



Intro to QCD: Hadron Spectroscopy, Scattering, the Lattice

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Modern Particle Physics

Four fundamental forces:

- Electromagnetic
- Weak
- Strong
- Gravity

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- ~~Gravity~~

Standard Model of Elementary Particles

	three generations of matter (fermions)			interactions / force carriers (bosons)	
	I	II	III		
mass	$\approx 2.2 \text{ MeV}/c^2$	$\approx 1.28 \text{ GeV}/c^2$	$\approx 173.1 \text{ GeV}/c^2$	0	$\approx 124.97 \text{ GeV}/c^2$
charge	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	0	0
spin	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	0
	u up	c charm	t top	g gluon	H de Higgs
	d down	s strange	b bottom	γ photon	
	e electron	μ muon	τ tau	Z Z boson	
	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	W W boson	

QUARKS (vertical label on the left side of the quark section)

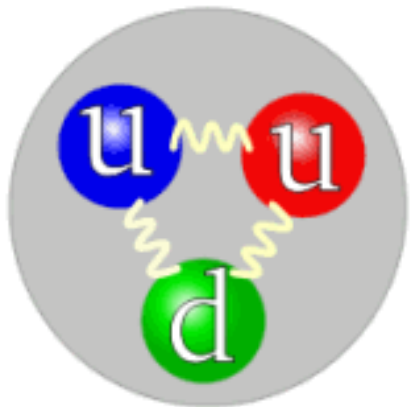
LEPTONS (vertical label on the left side of the lepton section)

GAUGE BOSONS VECTOR BOSONS (vertical label on the right side, red text)

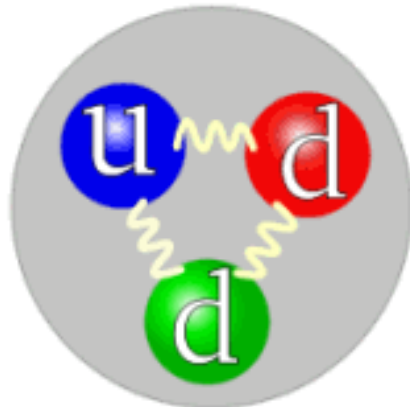
SCALAR BOSONS (vertical label on the right side, yellow text)

Quantum Chromodynamics:

- Fundamental degrees of freedom are 'quarks'
- Three charges (SU(3) color)
- Short range (~1fm)
- Very strong (~100x EM force)

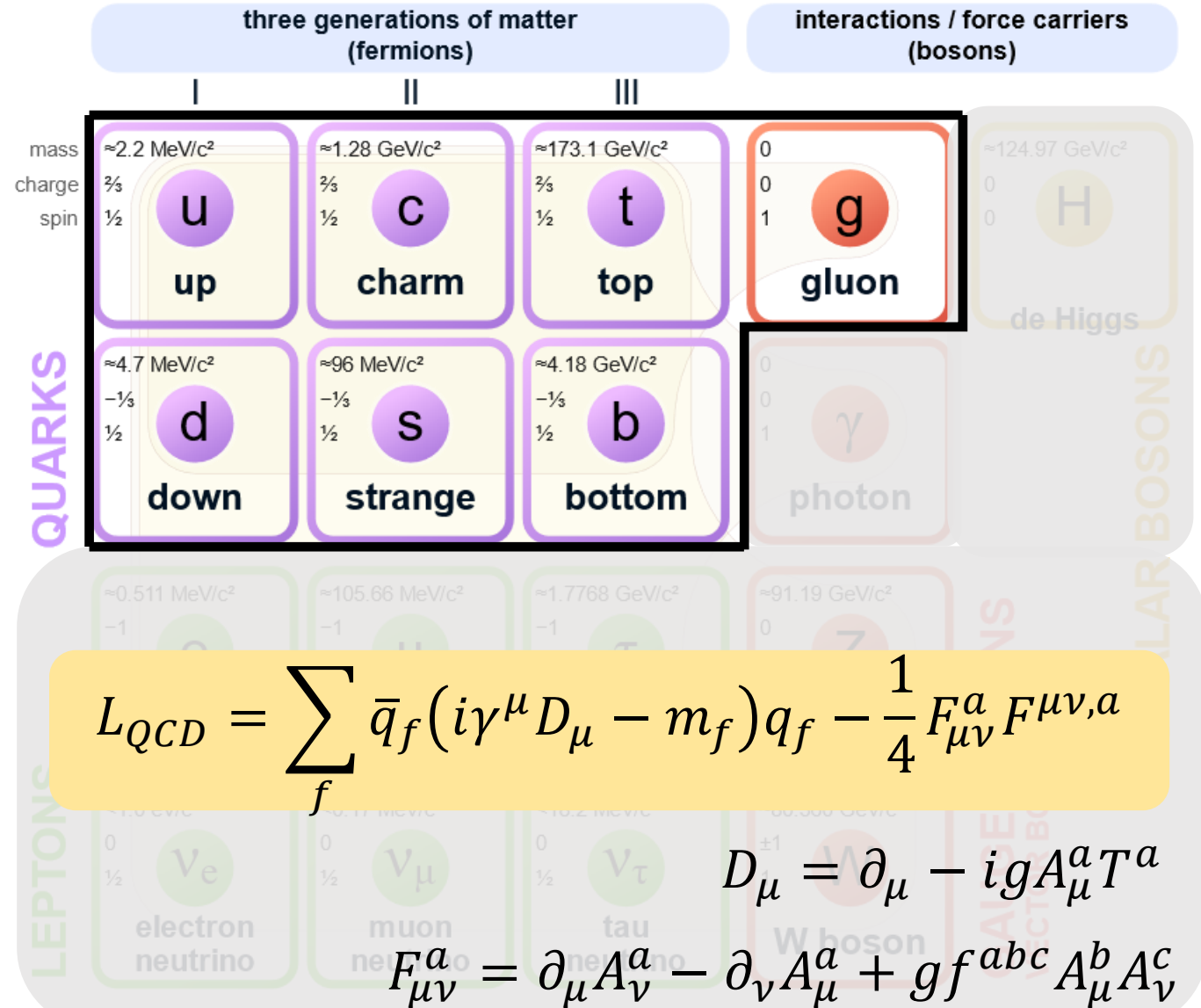


Proton



Neutron

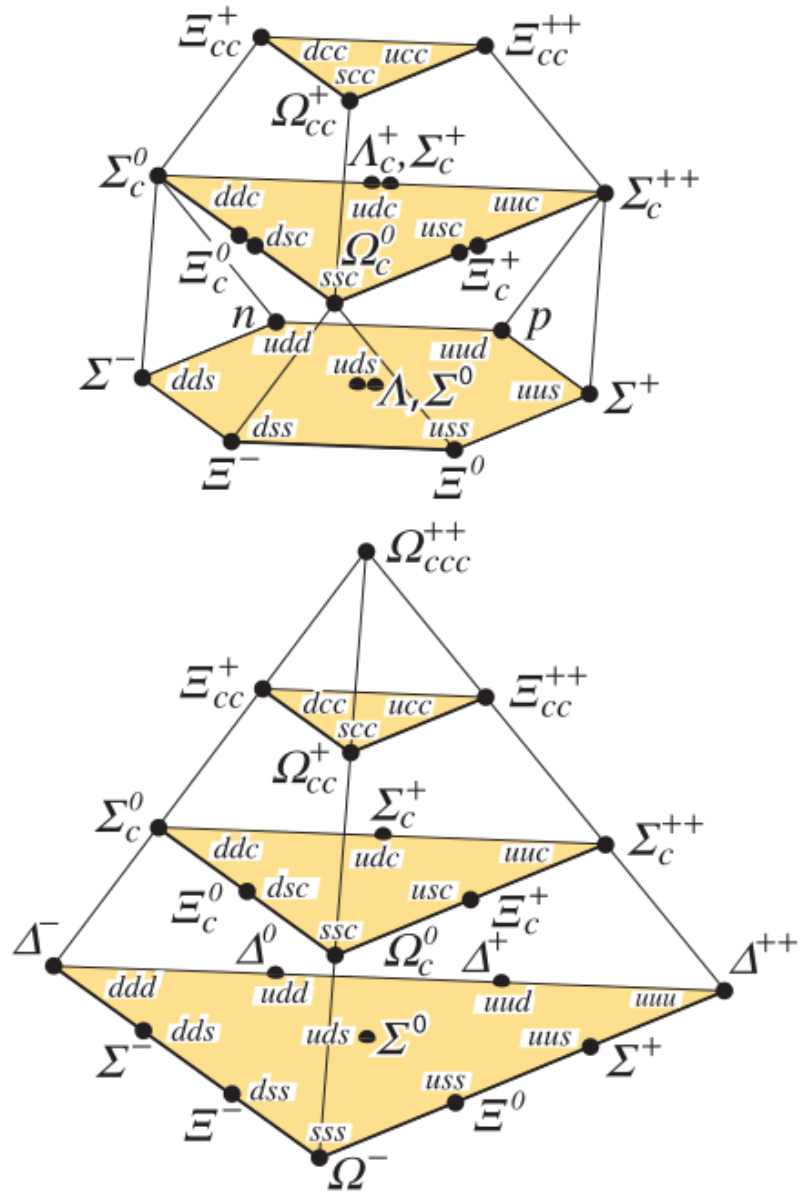
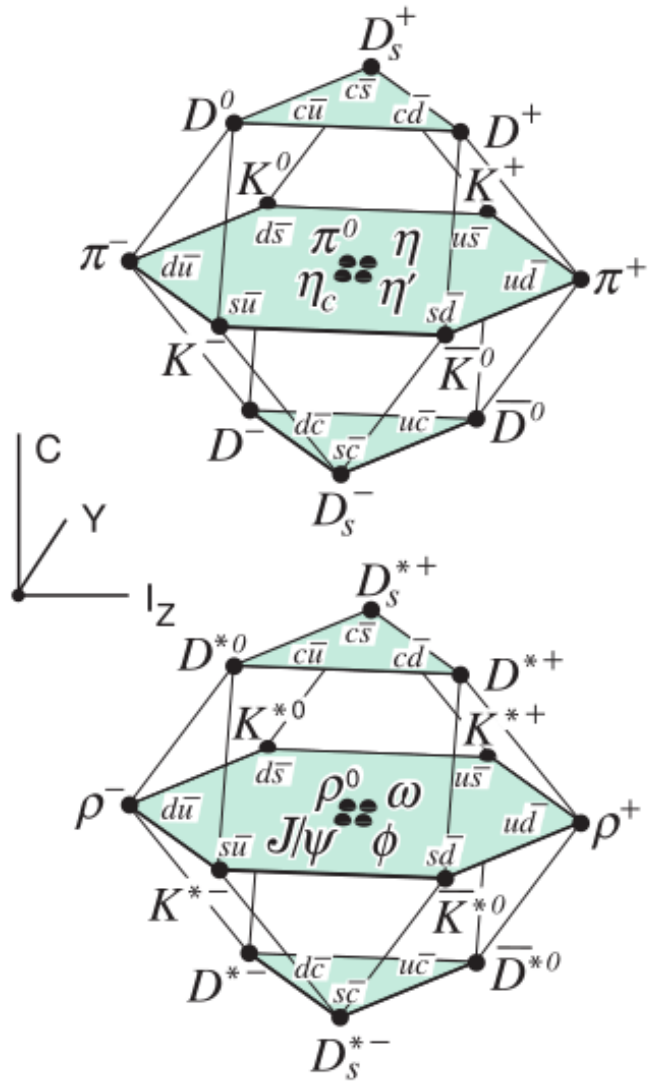
Standard Model of Elementary Particles



Quark model

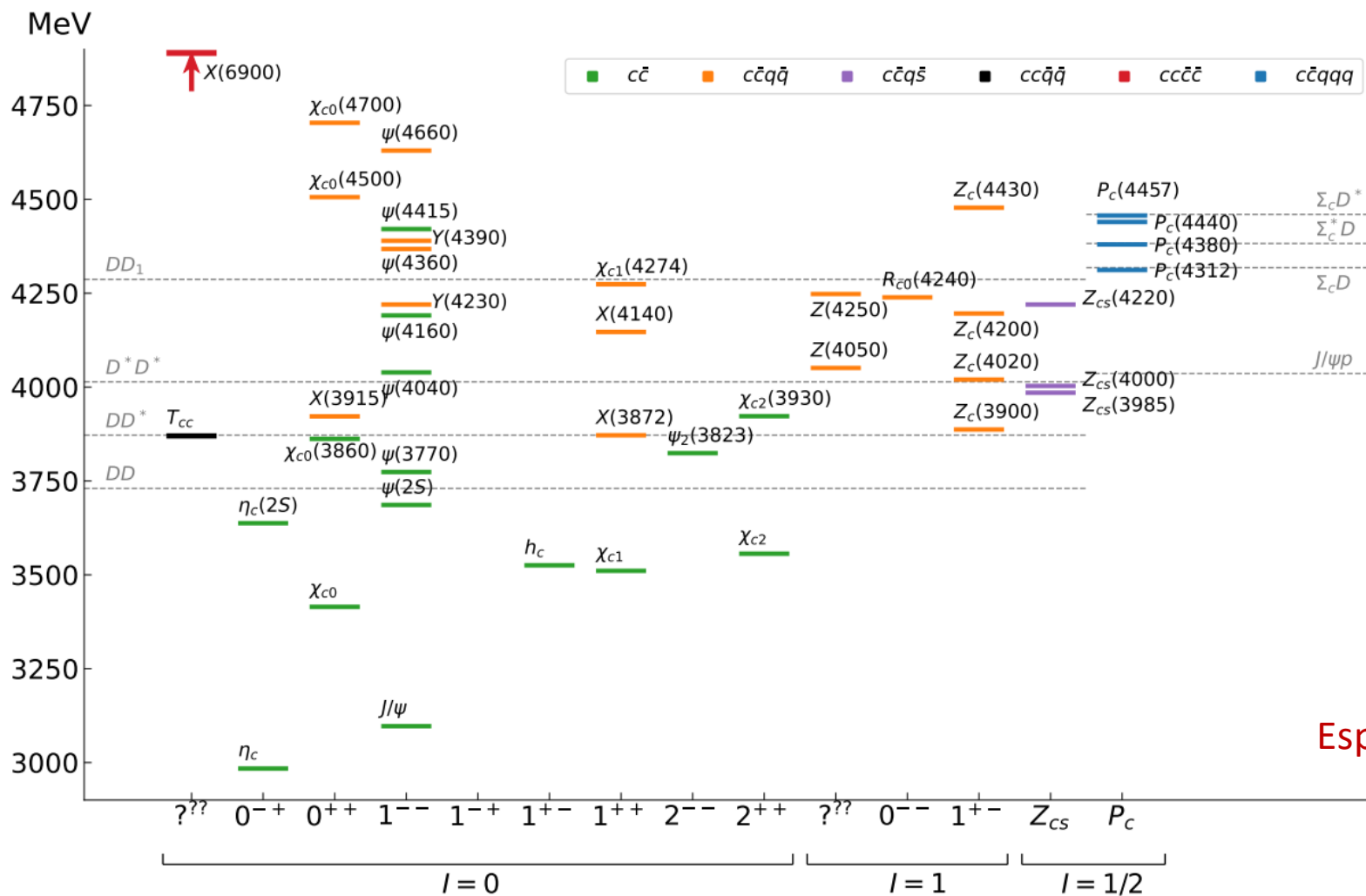
'Hadrons' are quark bound states:

- a) Mesons = $q\bar{q}$
- b) Baryons = qqq



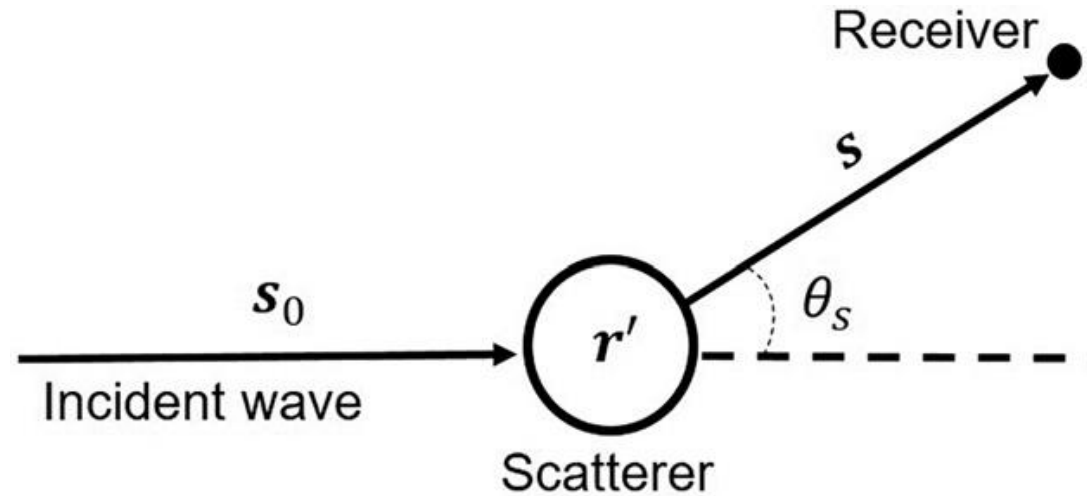
Exotics in $c\bar{c}$

Lots of new particles which are **incompatible** with usual $q\bar{q}/qqq$ description (J^{PC})



Esposito, AP, Polosa, Phys.Rept. 668
JPAC, arXiv:2112.13436

We match theory and expt. by calculating/measuring the 'Scattering Amplitude'



$$\frac{d\sigma}{d\Omega}(\theta, \phi) = |f(\theta, \phi)|^2$$

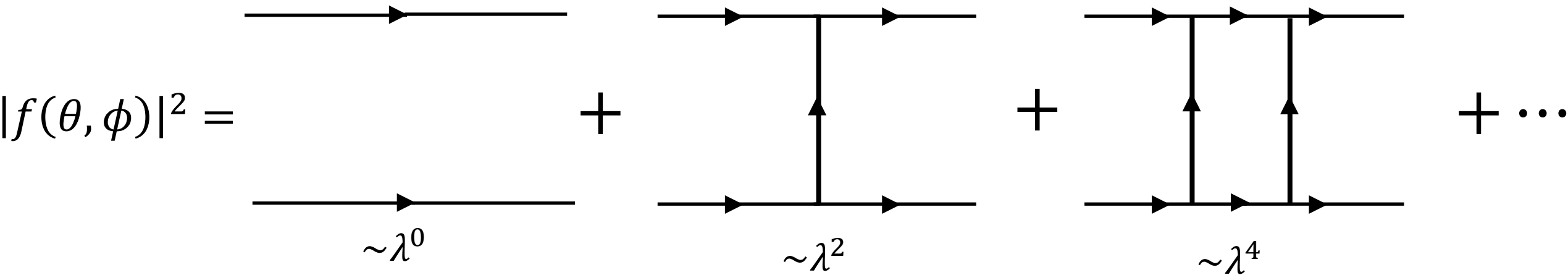
Differential cross section
(what expt. measures!)

Scattering amplitude
(Probability amplitude of
particle going in a direction)

Feynman Diagrams

$$L = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - \frac{1}{2} m \phi^2 - \frac{1}{3!} \lambda \phi^3$$

“phi-cubed theory”

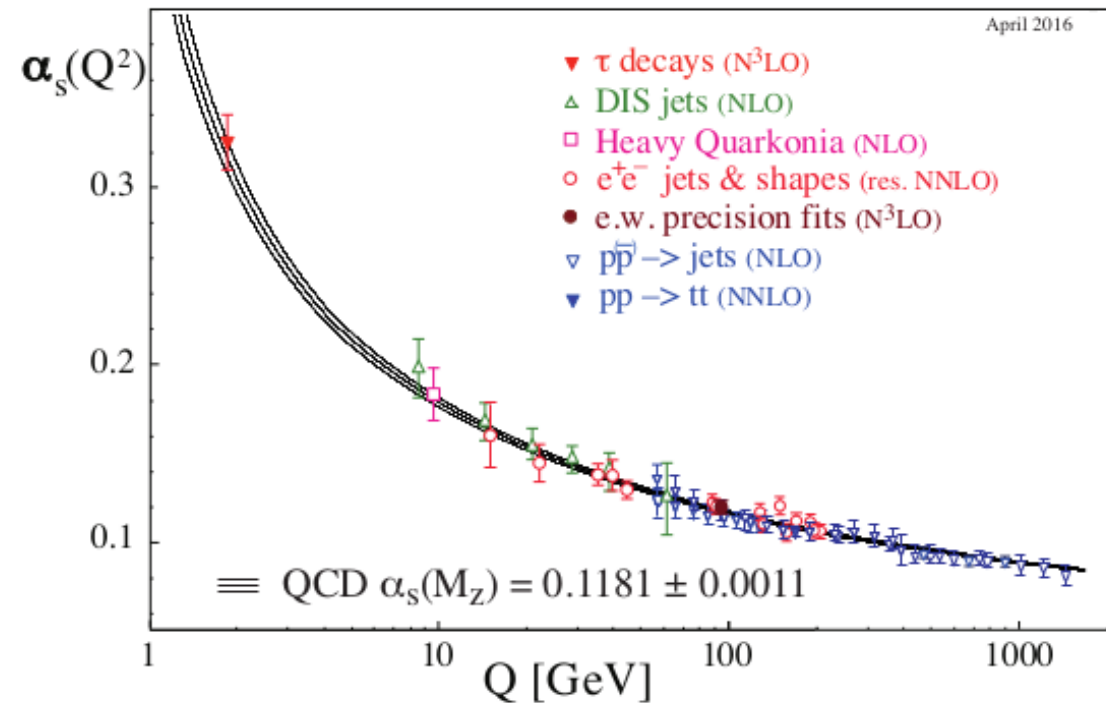
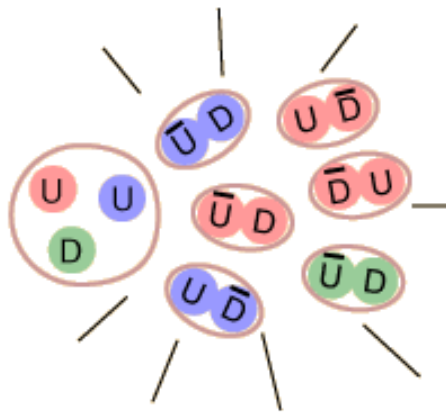
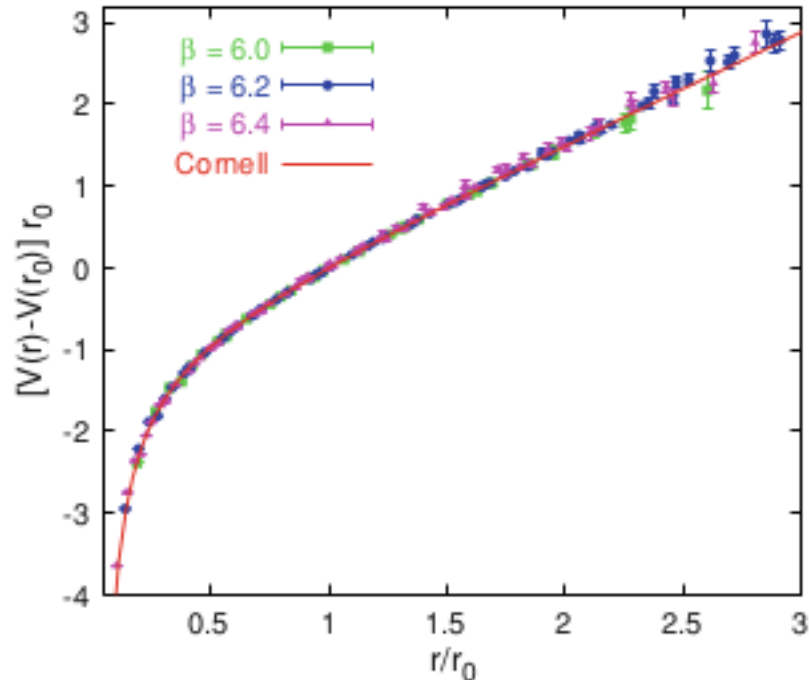


- Lines correspond to particle
- Vertices correspond to interactions of particles
- Perturbation Theory: Everything is a Taylor series!

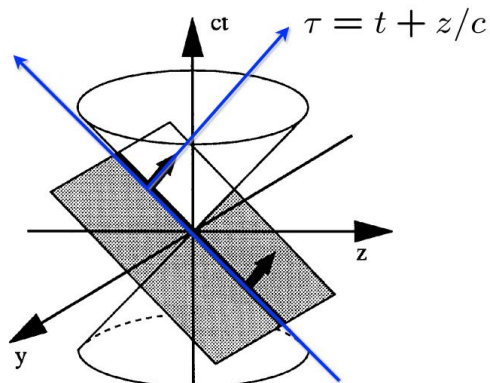
Problem: We can't solve QCD!

- QCD is non-abelian
- QCD is non-perturbative
- Confinement

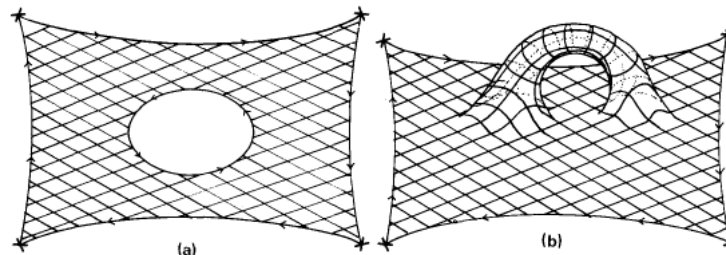
$$L_{QCD} = \sum_f \bar{q}_f (i\gamma^\mu D_\mu - m_f) q_f - \frac{1}{4} F_{\mu\nu}^a F^{\mu\nu,a}$$



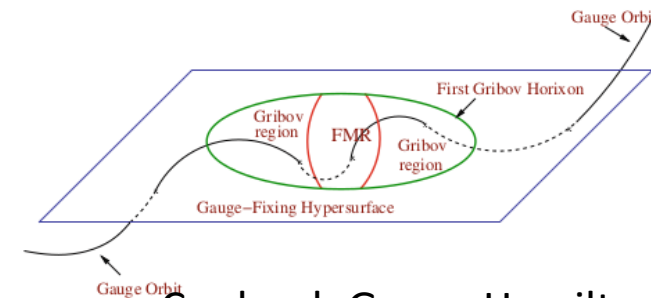
How do we study a theory we can't solve?



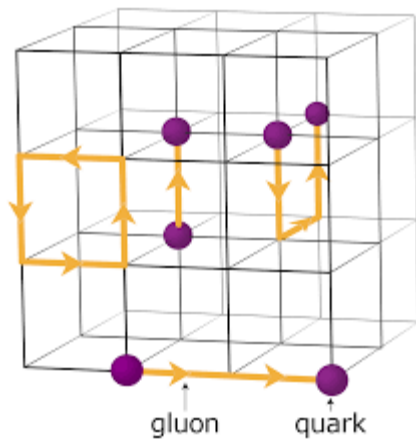
Light front



Large N Limit

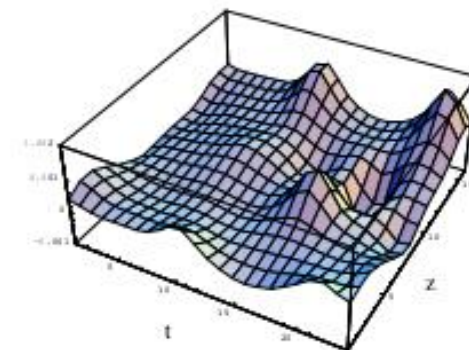


Coulomb Gauge Hamiltonian

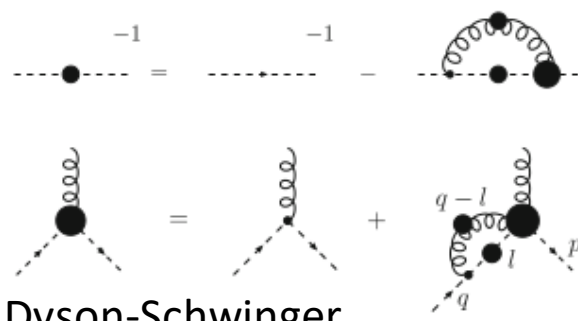


Lattice QCD

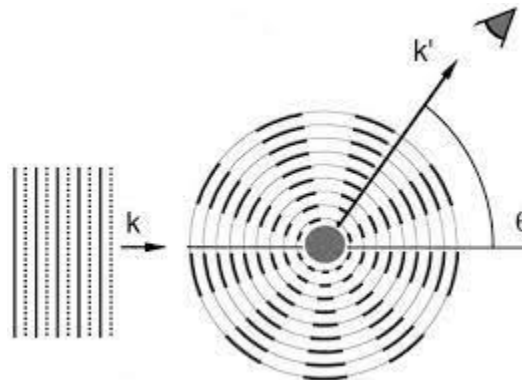
Confinement
QCD Spectrum



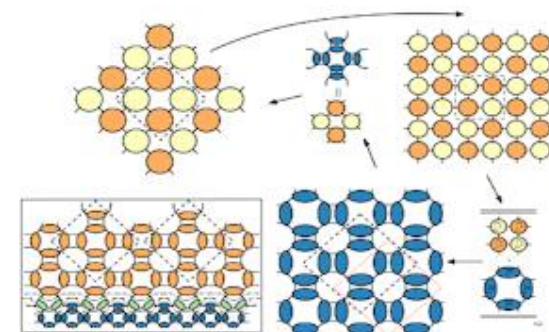
Solitons and Instantons



Dyson-Schwinger

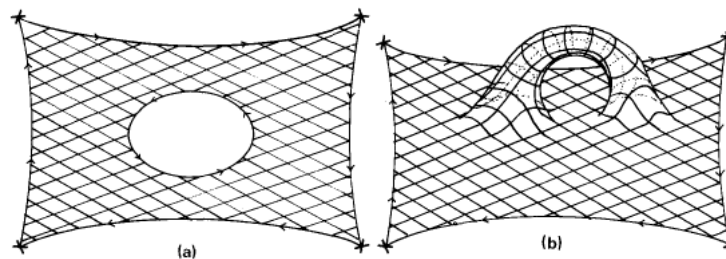
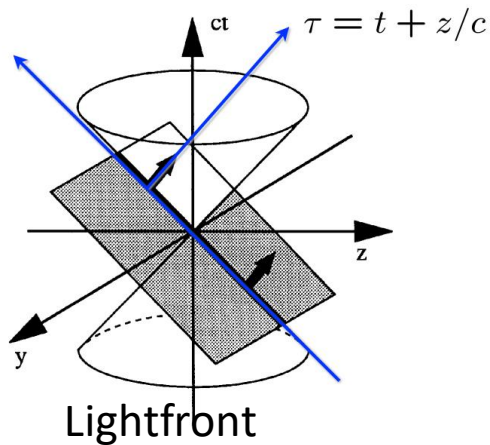


Scattering Theory

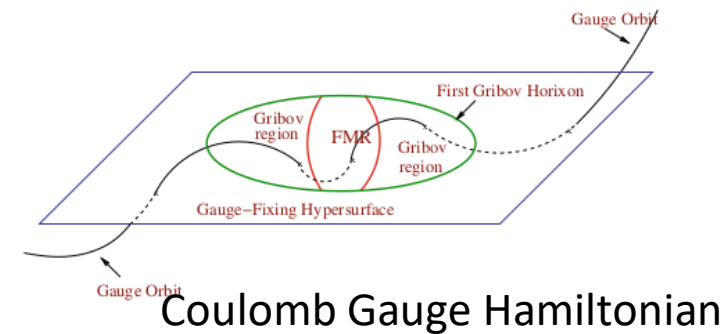


Tensor Networks

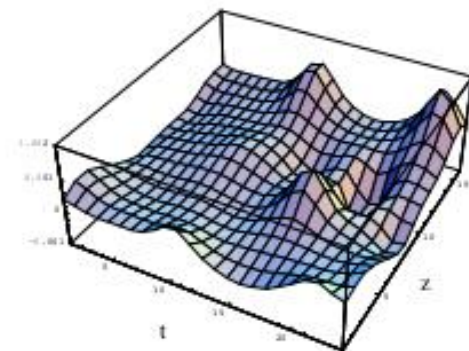
How do we study a theory we can't solve?



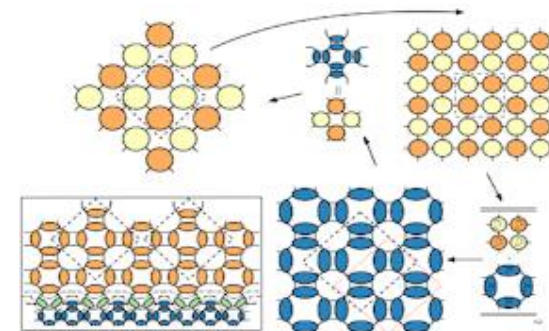
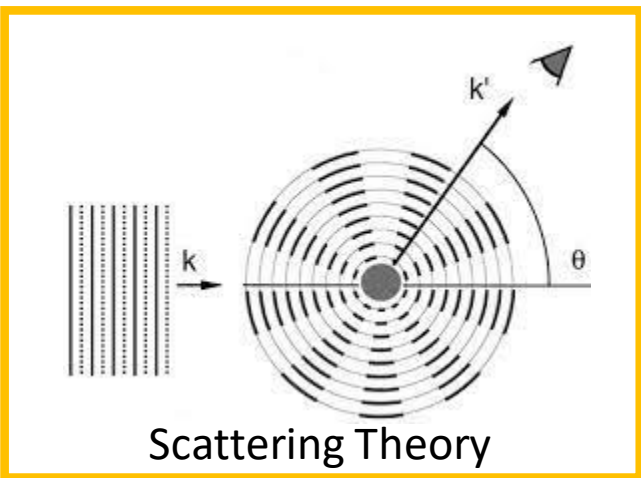
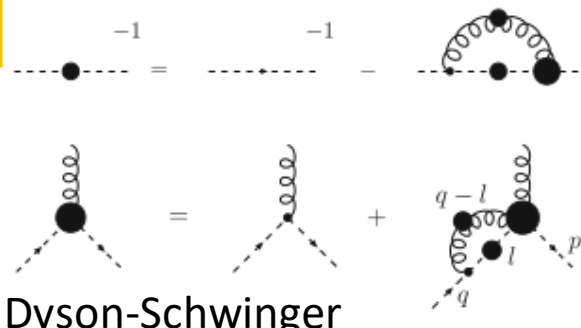
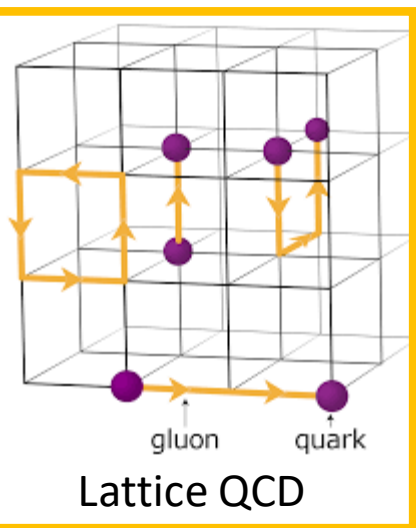
Large N Limit



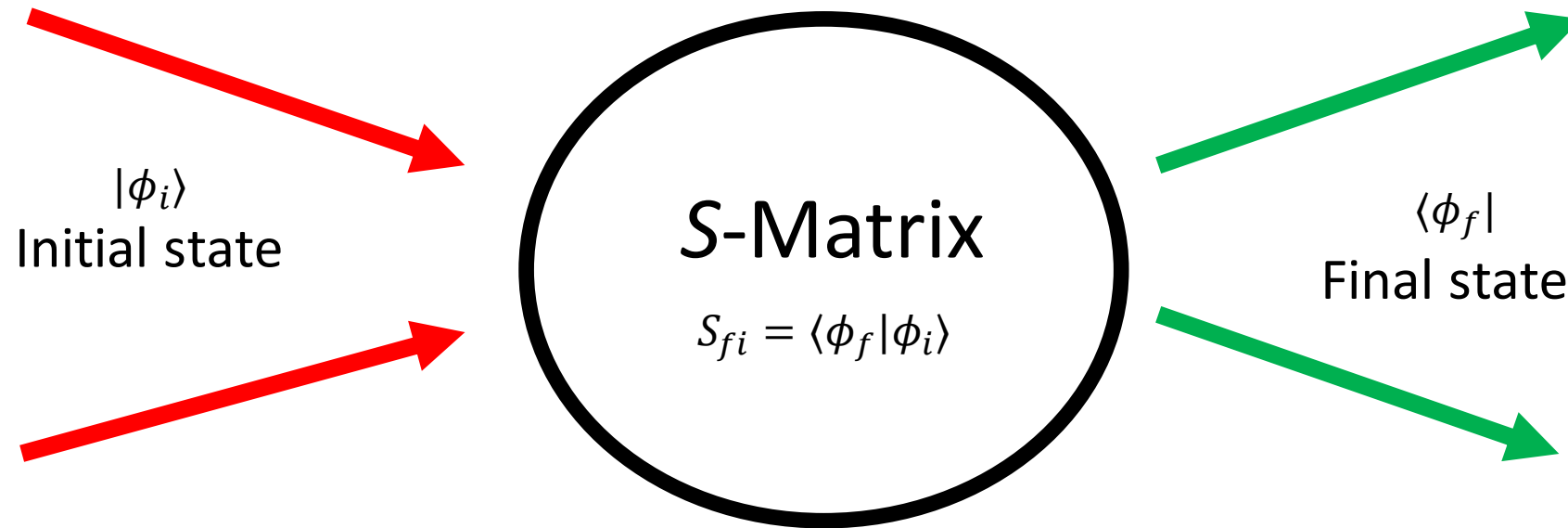
Confinement
QCD Spectrum



Solitons and Instantons

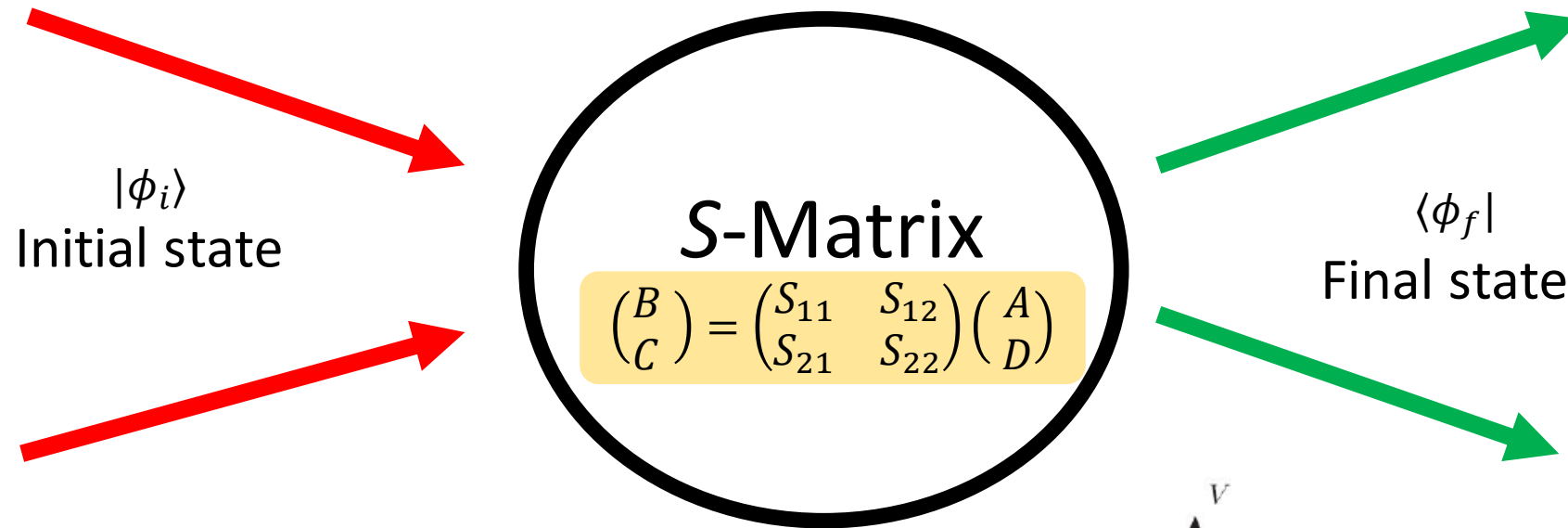


Scattering Theory: The S-Matrix

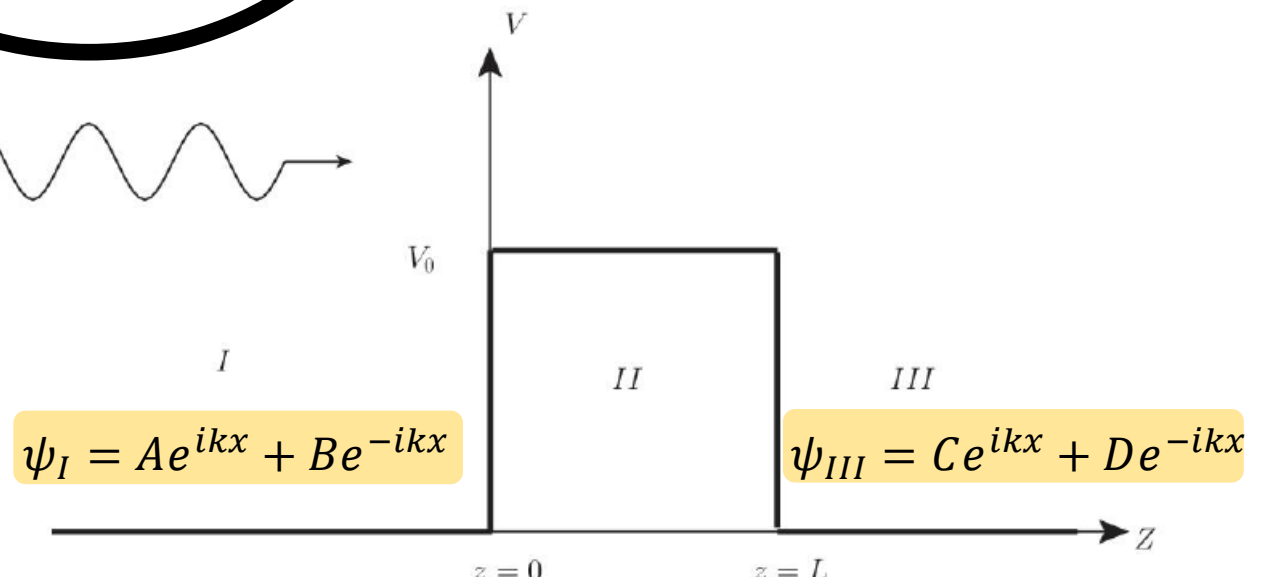


- Asymptotic states
- Interaction radius ($\sim 1\text{fm}$ for QCD)

Scattering Theory: The S-Matrix



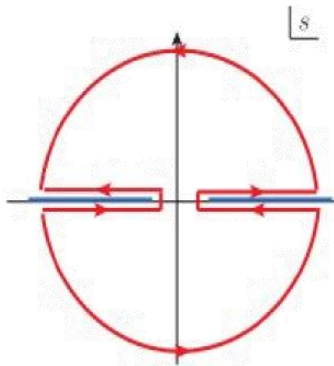
You've (probably) seen this before!



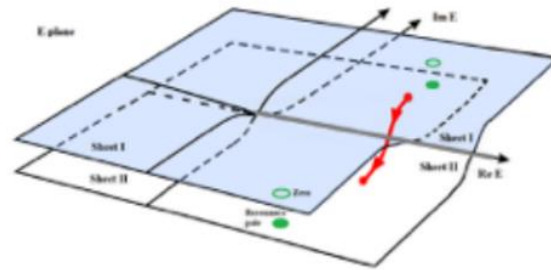
The S -Matrix principles

- Future cannot change the past (**analyticity**)
- 100%, something will happen (**unitarity**)
- The anti-particle is an anti-particle and not just a different particle (**crossing symmetry**)

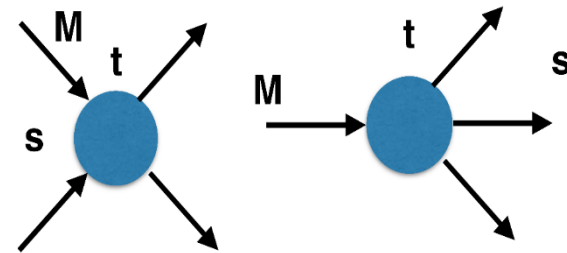
Sherlock Holmes, QFT



Analyticity



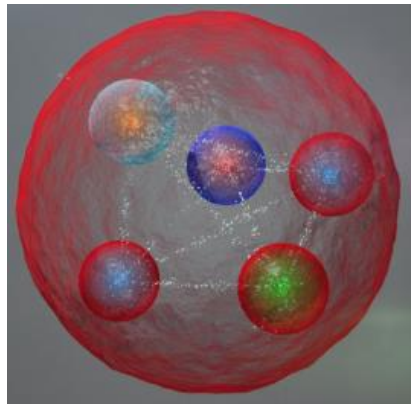
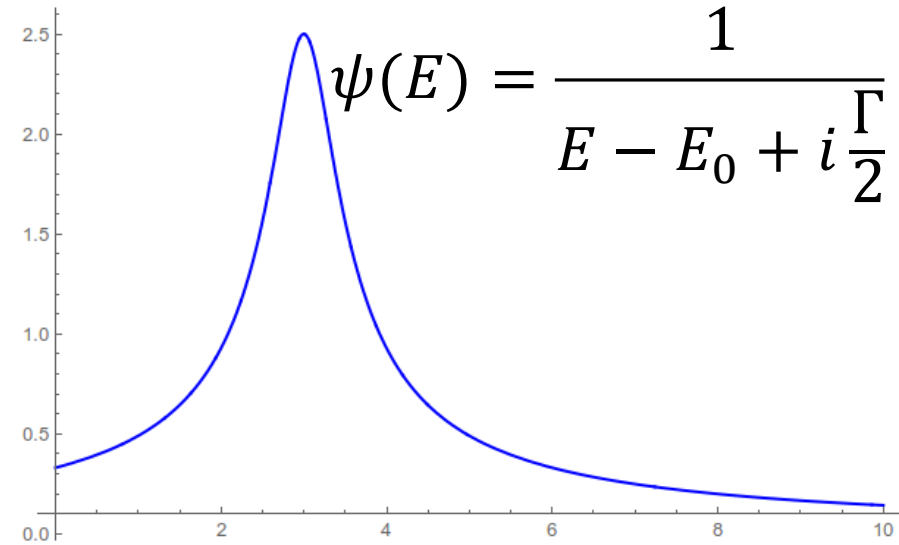
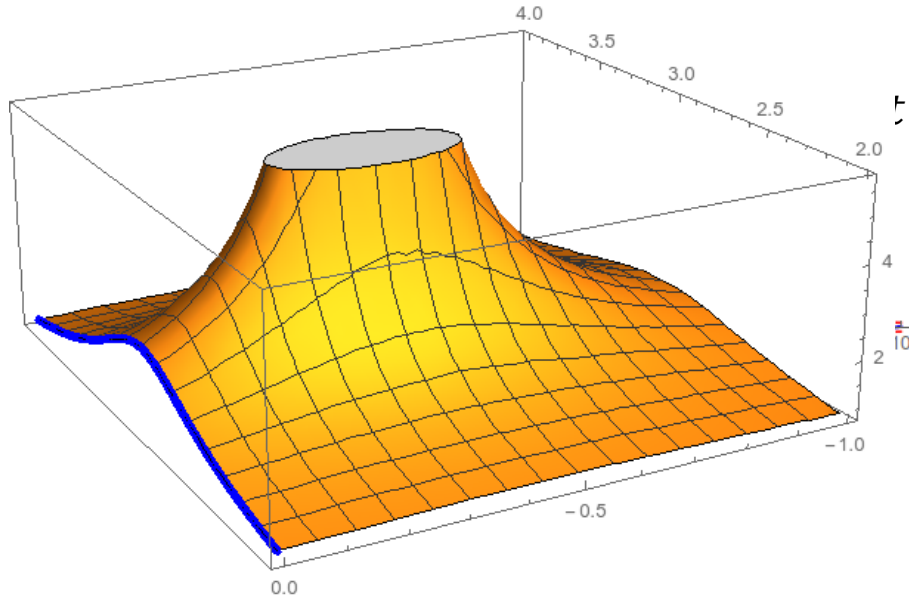
Unitarity



Crossing

Parameterize your ignorance. Build a model. Fit data. Have fun.

Resonances = Unstable states

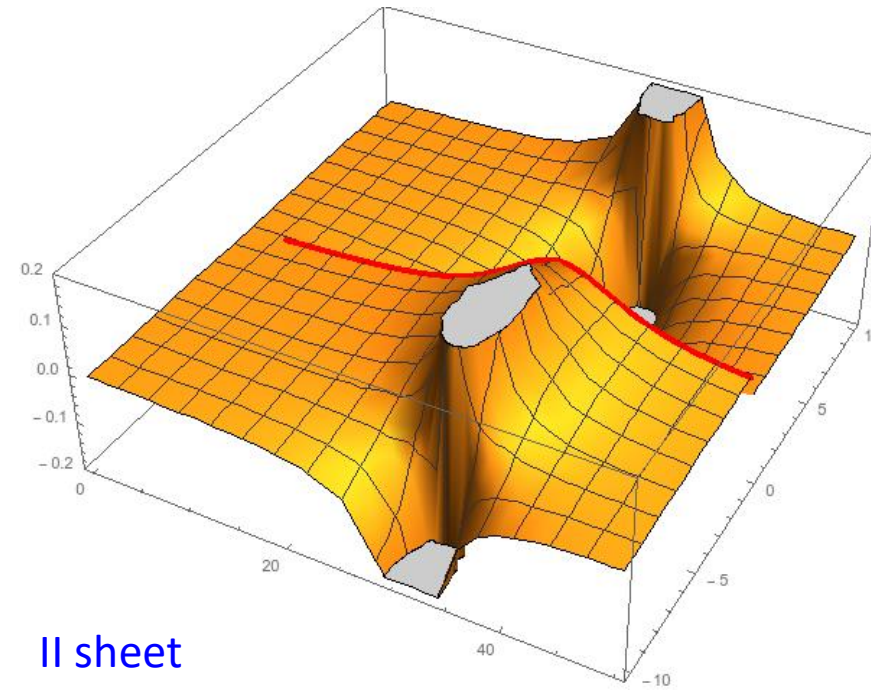
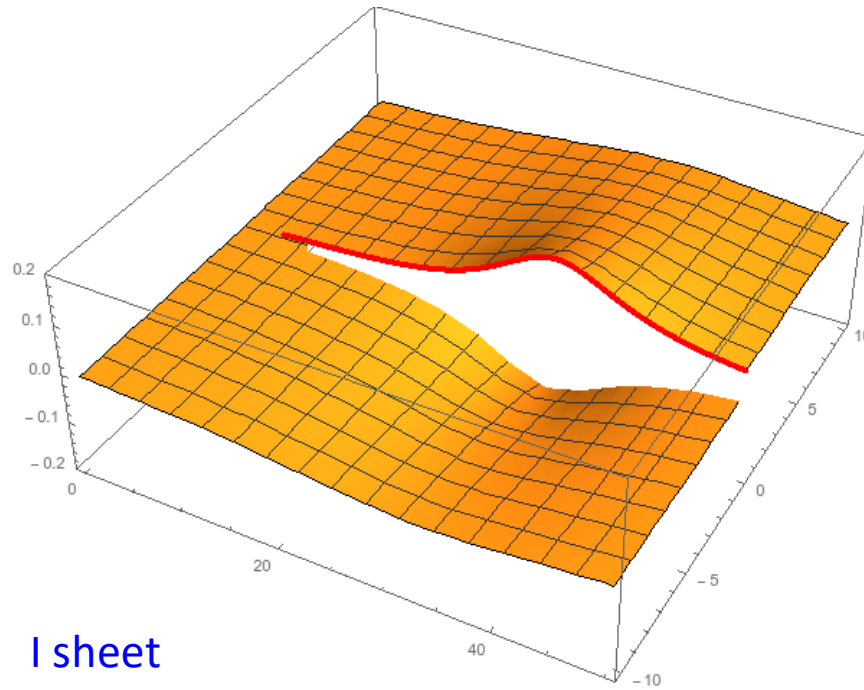


The function explodes for
 value of energy $E = E_0 + i\frac{\Gamma}{2}$
 Excited states are **unstable**

They manifest as **resonant peaks**
 in the complex energy plane

Unitarity & Pole hunting

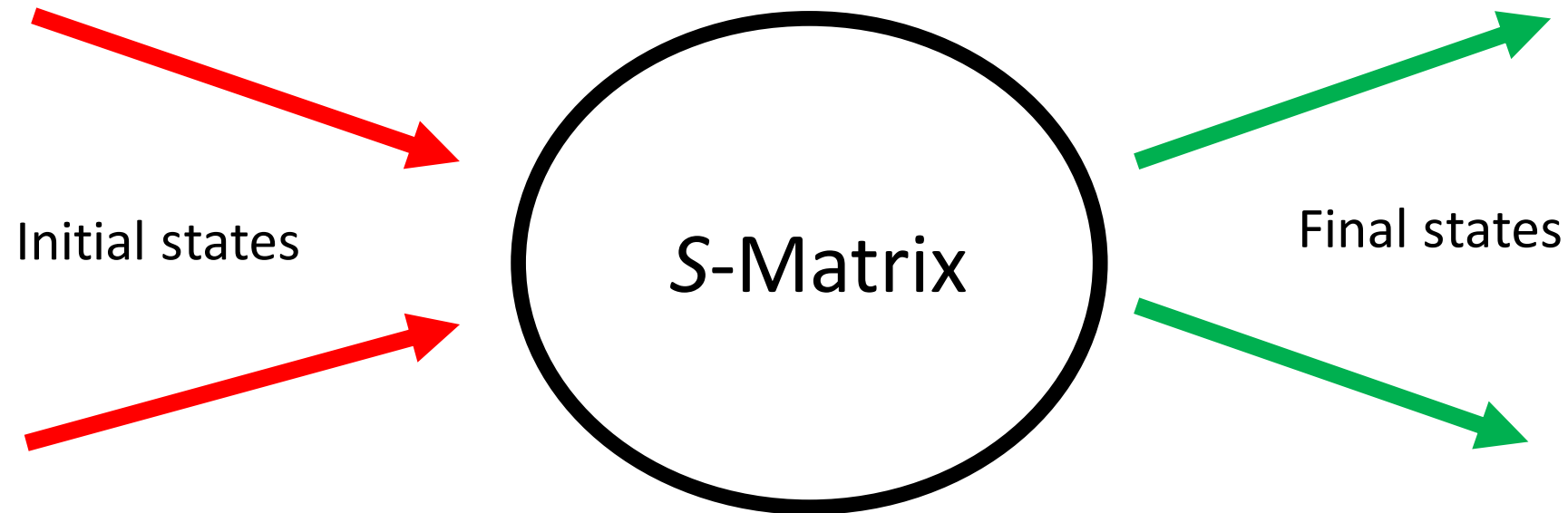
Unitarity creates a **branch cut** on the real axis,
two sheets continuously connected



Finding resonances means writing analytic amplitude,
and **hunting for poles** in the complex plane

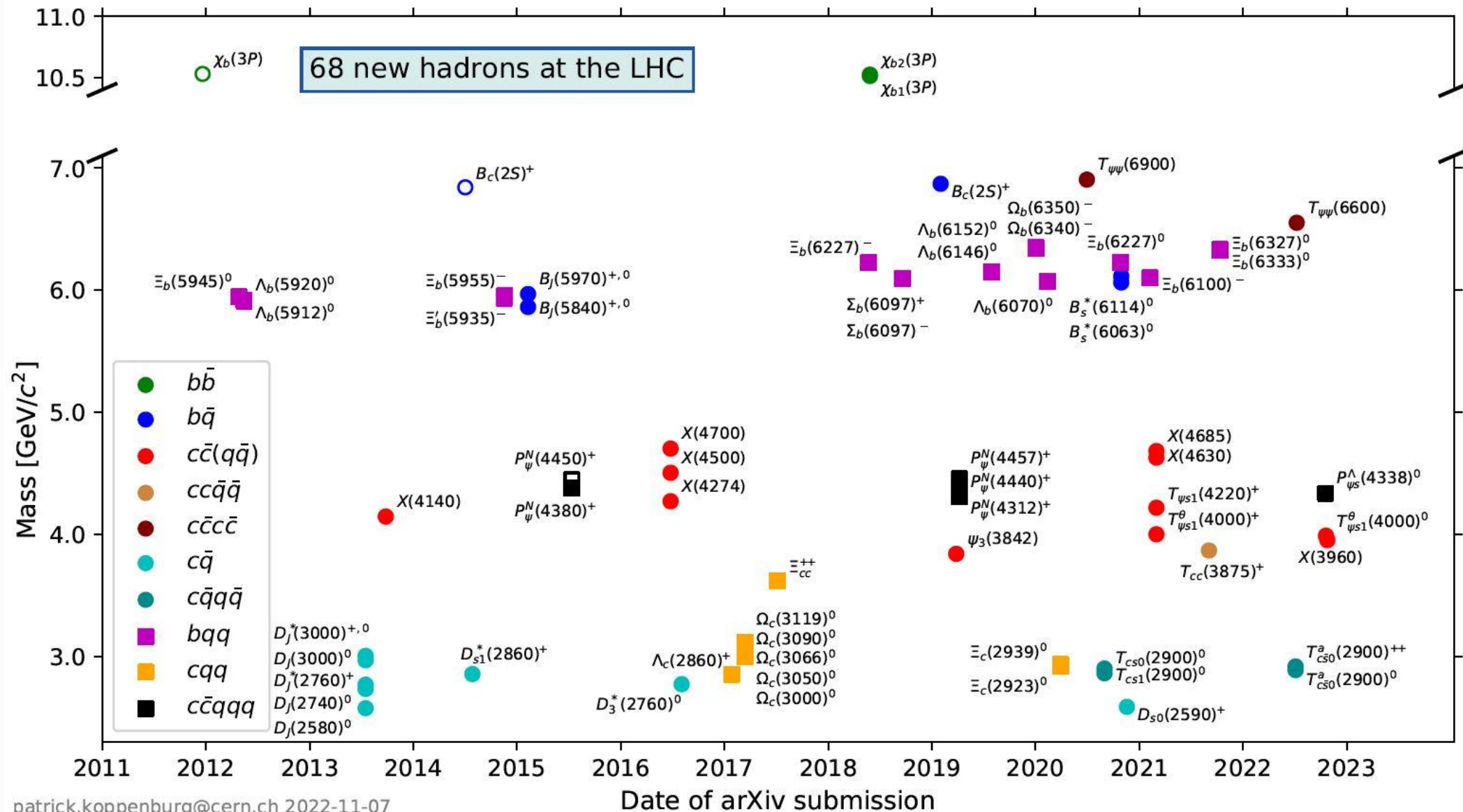
So what?

If we know the location of every pole, every resonance/bound state of particles in the S-matrix, we've **effectively solved QCD**



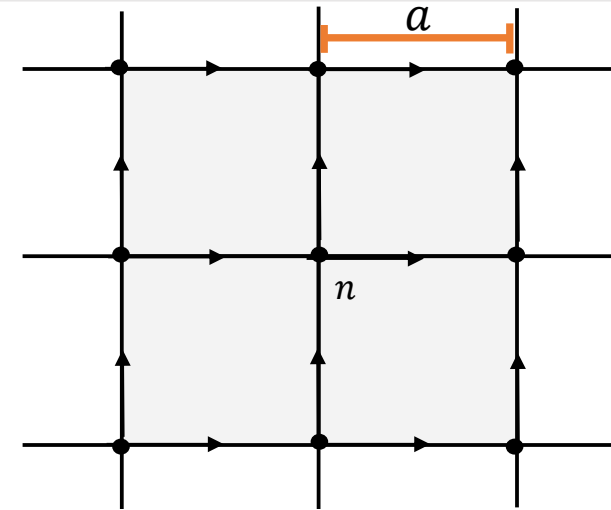
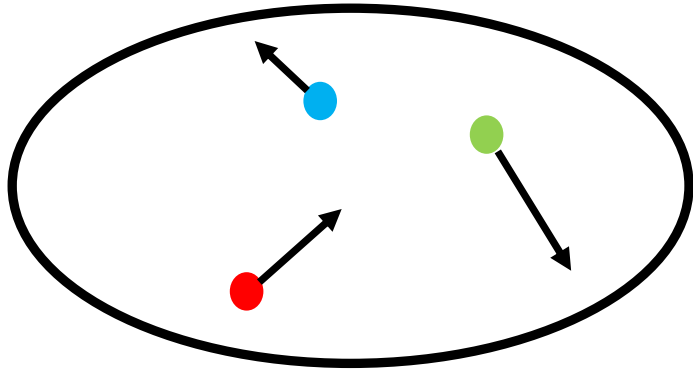
Just one problem...

Experimentalists keep finding more!



Lattice QCD

Basic Idea: Discretize spacetime, write QCD on a lattice, use a computer to calculate everything we want



$$U_\mu(n) = e^{iaA_\mu(n)}$$

$$L_{QCD} = \sum_f \bar{\psi}_f (i\gamma^\mu D_\mu - m_f) \psi_f - \frac{1}{4} F_{\mu\nu}^a F^{\mu\nu,a}$$

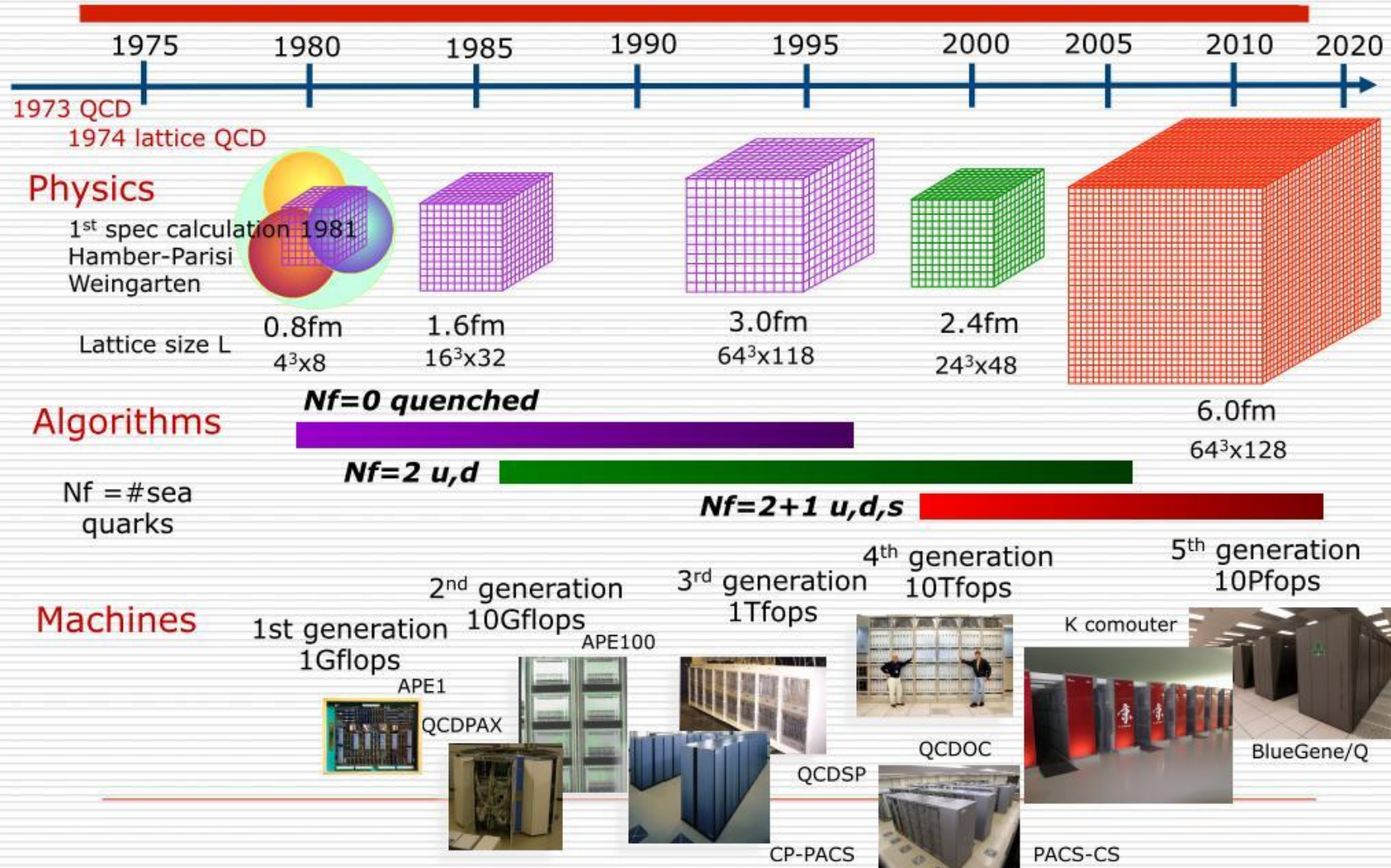
$$S_{Lat} = \frac{\beta}{3} \sum_n \sum_{\mu < \nu} \text{Re Tr}[1 - U_{\mu\nu}(n)]$$

- No approximations required!
- Systematically improvable
- Gives access to quark operators
- More 'experimental' data to use to check models

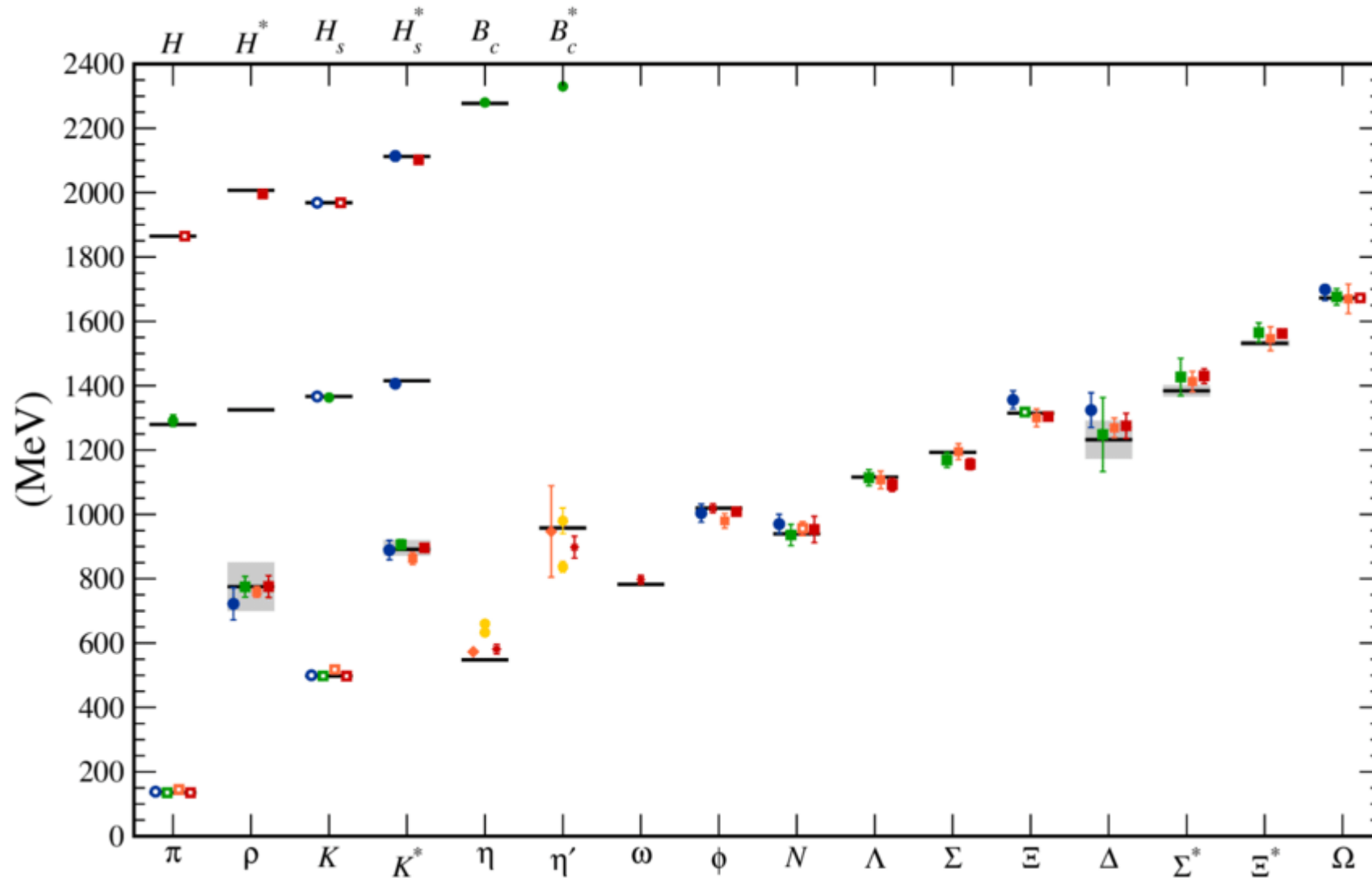
- Very computationally expensive (fermions = pain)
- Need to write observables in terms of 'links'



Four decades of Lattice QCD

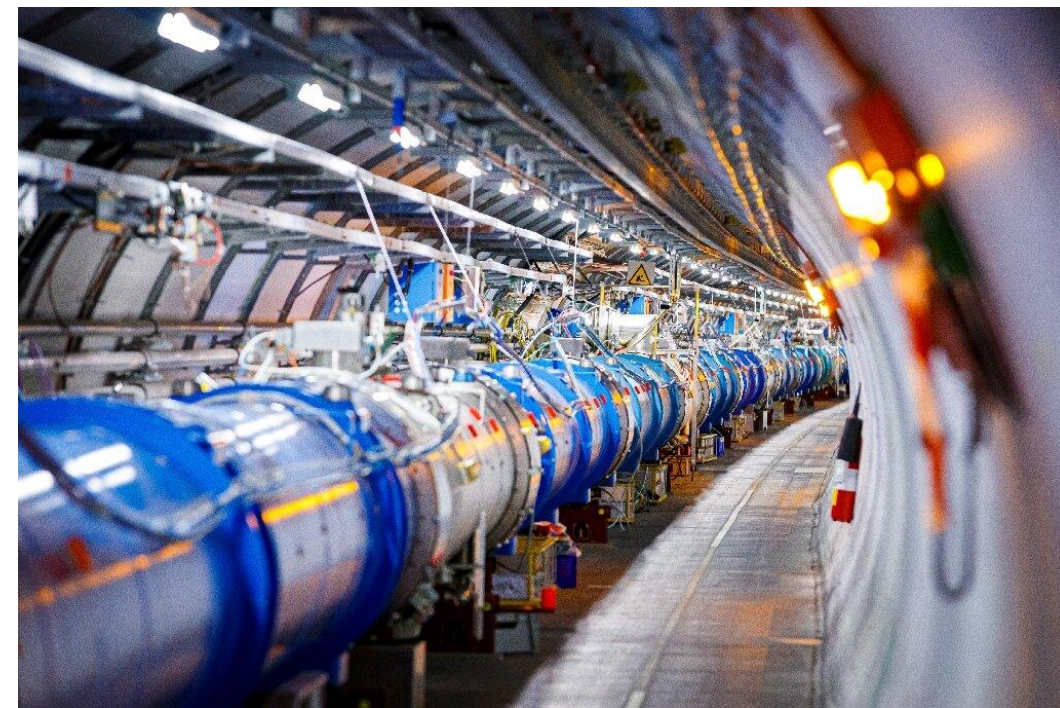


Hadrons from LQCD

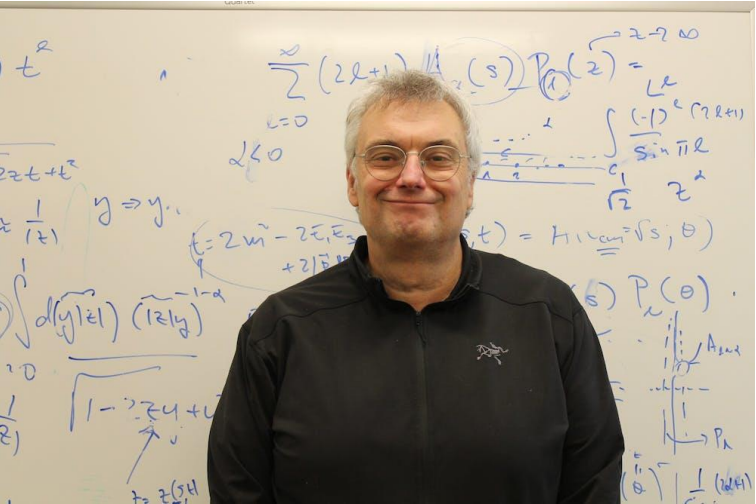


Summary

- QCD is difficult to deal with but not impossible
- S-matrix principles constrain the physics
- Theory, Lattice and Expt. work together to make sense of the hadron spectrum



Bonus: Grad School at IU



- Choose a school based on prospective advisors!
- Are the students happy?
- Travel = 😊
- \$\$\$ = 😊
- Grad Worker Union = \$\$\$

