

Towards the hybrid meson photoproduction at JLab: unraveling pion exchange from a Regge theory perspective

Glòria Montaña Faiget

Theory Center, Thomas Jefferson National Accelerator Facility

In collaboration with A. Szczepaniak, V. Mathieu and others

Nuclear Theory Seminar

April 18, 2024

Confined states of quarks and gluons

Mesons and baryons aren't the only states allowed by QCD.

A SCHEMATIC MODEL OF BARYONS AND MESONS *

M. GELL-MANN
 California Institute of Technology, Pasadena, California

Received 4 January 1964

... Baryons can now be constructed from quarks by using the combinations (qqq), (qqqq \bar{q}), etc., while mesons are made out of (q \bar{q}), (qq $\bar{q}\bar{q}$), etc. ...

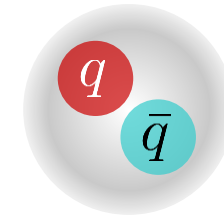
[M.Gell-Mann, *Phys.Lett.* 8 (1964) 214]

AN SU_3 MODEL FOR STRONG INTERACTION SYMMETRY AND ITS BREAKING

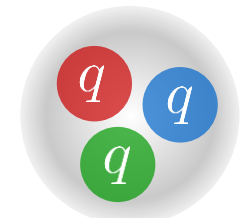
G. Zweig *)
 CERN - Geneva

In general, we would expect that baryons are built not only from the product of three aces, AAA, but also from $\bar{A}AAAA$, $\bar{A}\bar{A}AAAA$, etc., where \bar{A} denotes an anti-ace. Similarly, mesons could be formed from $\bar{A}A$, $\bar{A}\bar{A}AA$ etc. For the low mass mesons and baryons we will assume the simplest possibilities, $\bar{A}A$ and AAA, that is, "deuces and treys".

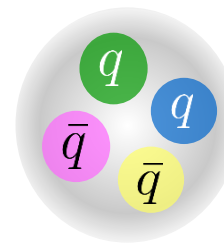
[G.Zweig, CERN-TH-401 (1964)]



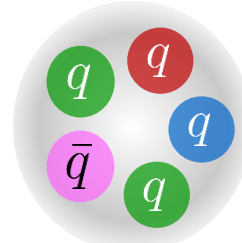
mesons



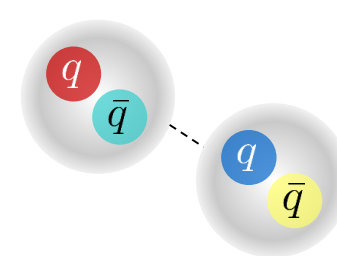
baryons



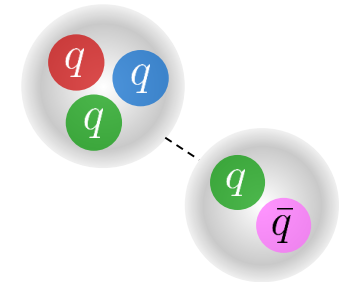
tetraquarks



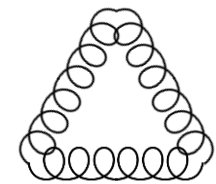
pentaquarks



hadronic molecules



hybrid mesons



glueballs

Hybrid mesons with exotic quantum numbers

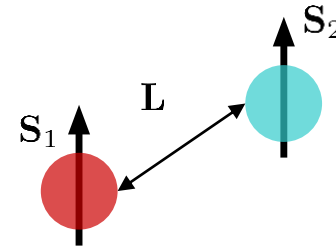
- Mesons are experimentally characterized by quantum numbers:

→ Isospin

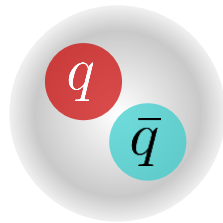
→ Total angular momentum $\mathbf{J} = \mathbf{L} + \mathbf{S}$ ($\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2$)

→ Parity $P = -(-1)^L$

→ Charge conjugation $C = (-1)^{L+S}$



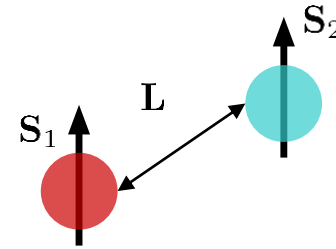
Conventional mesons



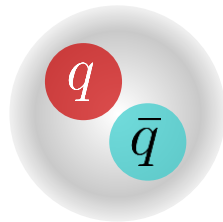
J^{PC}	}		0^{++}	0^{-+}	
		1^{--}	1^{++}		1^{+-}
		2^{--}	2^{++}	2^{-+}	
		3^{--}	3^{++}		3^{+-}
		\vdots	\vdots	\vdots	\vdots

Hybrid mesons with exotic quantum numbers

- Mesons are experimentally characterized by quantum numbers:
 - Isospin
 - Total angular momentum $\mathbf{J} = \mathbf{L} + \mathbf{S}$ ($\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2$)
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Conventional mesons



J^{PC}

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Hybrid mesons

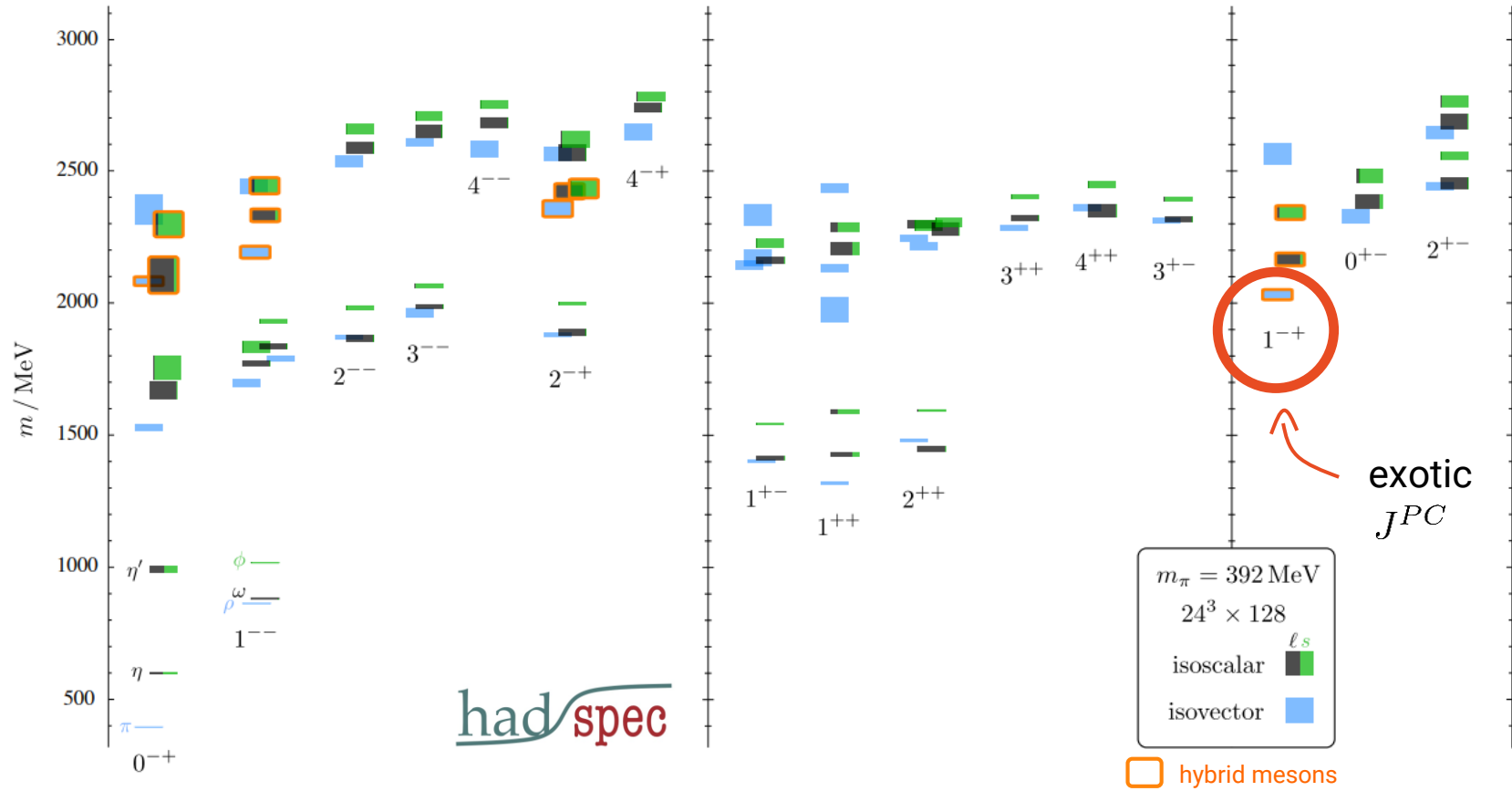


- Gluonic fields in hybrid mesons give rise to states with **“exotic” quantum numbers**

[C.A.Meyer and Y.Van Haarlem, *Phys.Rev.C* 82 (2010) 025208]

↪ “Smoking gun”

Spectrum of light mesons from lattice QCD

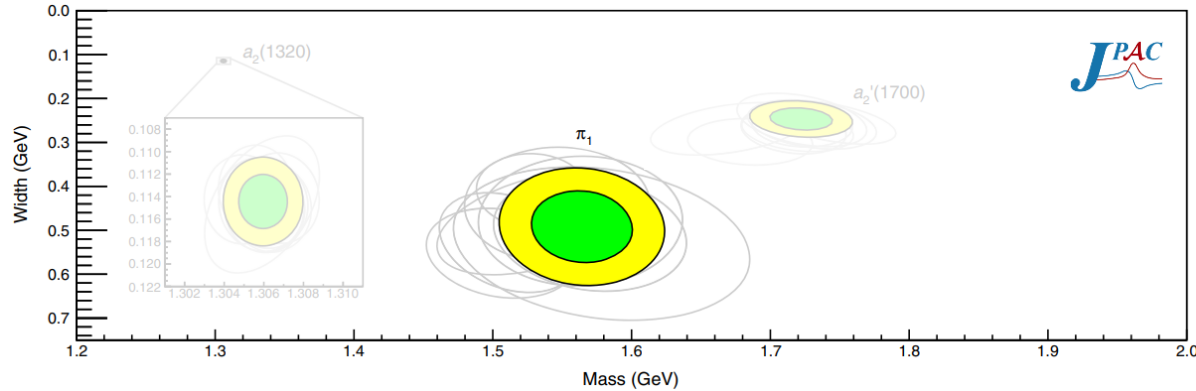


[J.J.Dudek, R.G.Edwards, P.Guo, and C.E.Thomas, *Phys.Rev.D* 88 (2013) 9, 094505]

Search for exotic hybrid mesons

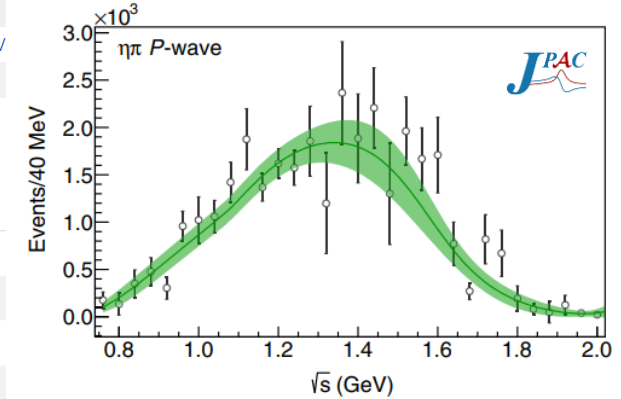
- Best evidence for a hybrid meson is for π_1 in pion-production at COMPASS.
- Two 1^{-+} isovector states in the PDG.
- Coupled channel analyses favor existence of only one broad π_1 state consistent with $\pi_1(1600)$ in the 1400–1600 MeV region.

[A.Rodas, et al. (JPAC), Phys.Rev.Lett. 122 4, 042002 (2019)]

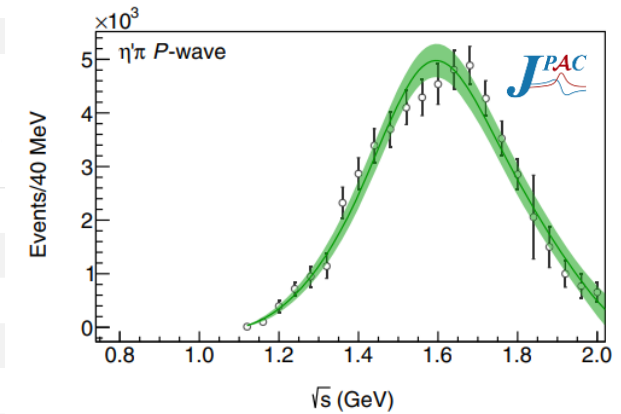


Data:
C.Adolph et al.,
Phys.Lett.B 740, 303 (2015)

$\pi_1(1400)$	$I^G(J^{PC}) = 1^-(1^{-+})$
$\pi_1(1400)$ T-MATRIX POLE \sqrt{s}	$(1405 \pm 4^{+15}_{-18}) - i(314 \pm 14^{+18}_{-69})$ MeV
$\pi_1(1400)$ MASS	1354 ± 25 MeV ($S = 1.8$)
$\pi_1(1400)$ WIDTH	330 ± 35 MeV
Decay Modes	
Mode	Fraction (Γ_i / Γ)
Γ_1 $\eta\pi^0$	seen
Γ_2 $\eta\pi^-$	seen
Γ_3 $\eta'\pi$	
Γ_4 $\rho(770)\pi$	not seen



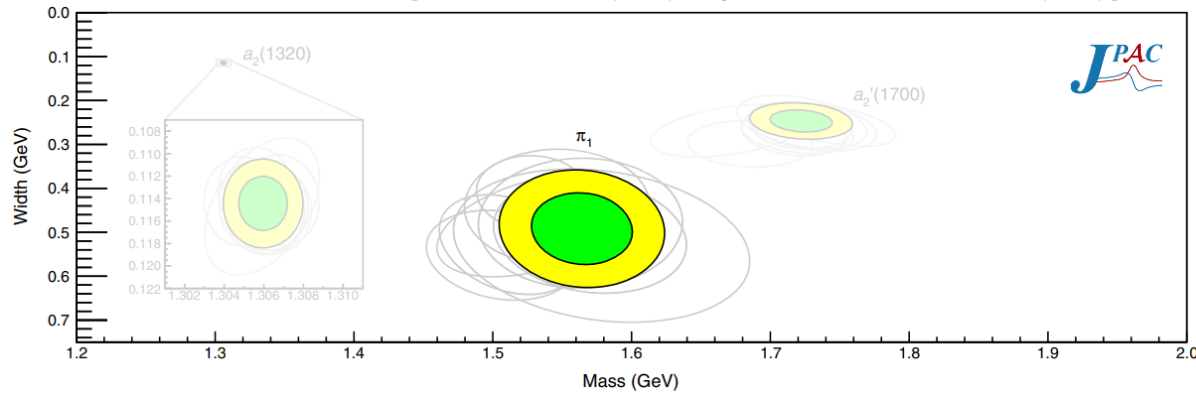
$\pi_1(1600)$	$I^G(J^{PC}) = 1^-(1^{-+})$
$\pi_1(1600)$ T-Matrix Pole \sqrt{s}	$(1480 - 1680) - i(150 - 300)$ MeV
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Γ_5 $\eta'(958)\pi^-$	seen
Γ_6 $\eta\pi$	seen
Γ_7 $f_1(1285)\pi$	seen



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- Recent evidence for η_1 and η_1' from BES-III.

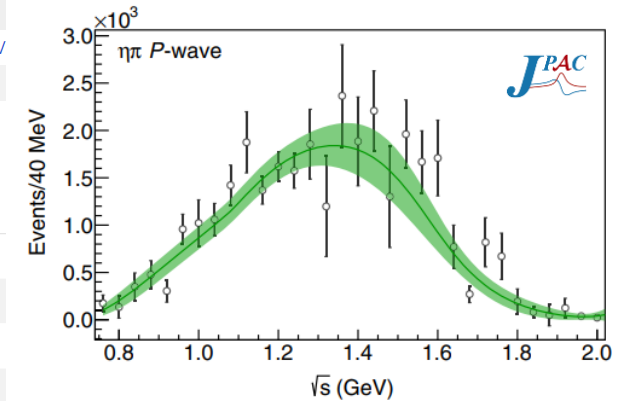
[M.Ablikim et al., *Phys.Rev.Lett.* 129 (2022) 19]

- Need to confirm π_1 and $\eta_1^{(\prime)}$ and establish the light quark hybrid spectrum.

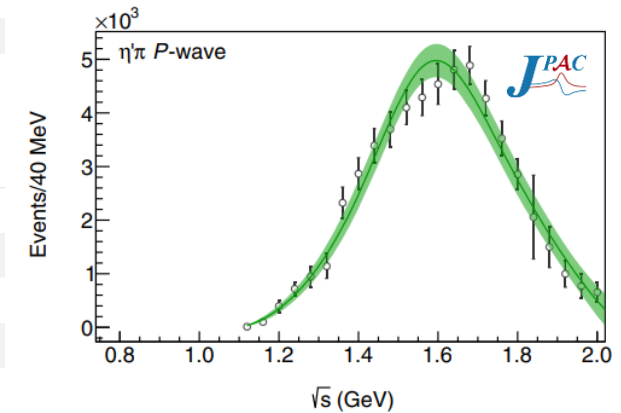


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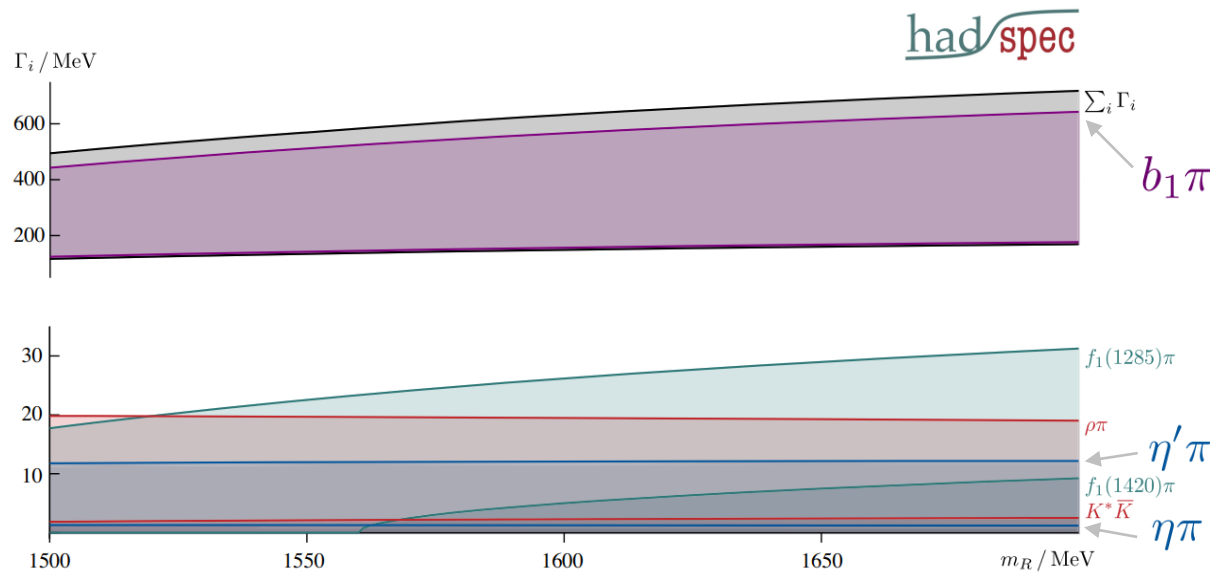


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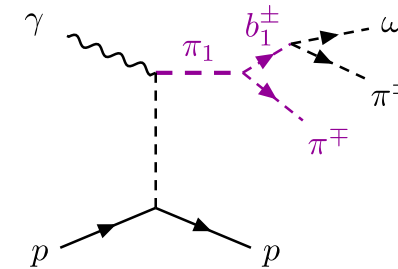


Search for exotic hybrid mesons in photoproduction at JLab

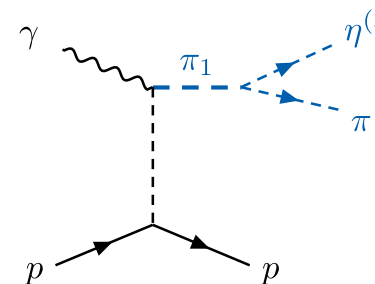
- Identifying the spectrum of hybrid mesons in photoproduction is the primary purpose of the **GLUEX** experiment.
- Exotic hybrid cross-sections ($S_{q\bar{q}} = 1$) expected to be enhanced with photon beam.
- Experimentally challenging: production + decay.
- Lattice QCD calculations suggest $b_1\pi$ is the dominant decay channel of $\pi_1(1600)$.



[A.J.Woss, J.J.Dudek, R.G.Edwards, C.E.Thomas and D.J.Wilson, *Phys.Rev.D* 103 5, 054502 (2021)]



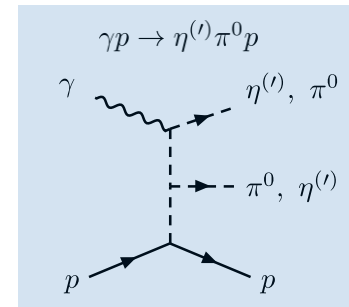
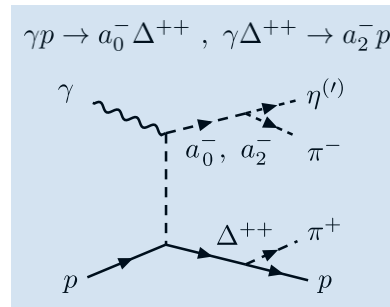
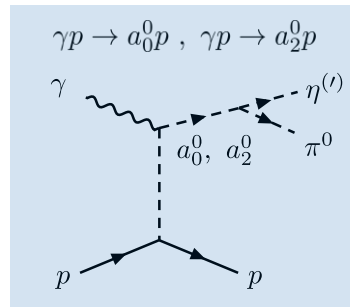
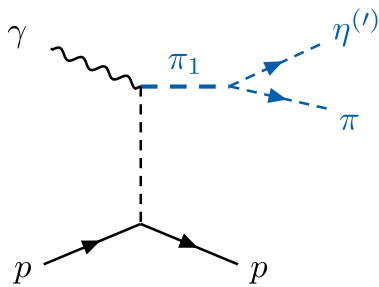
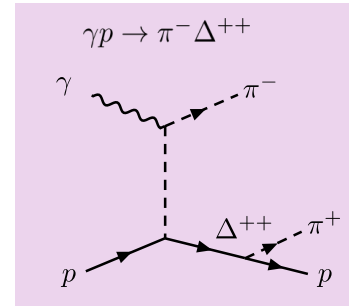
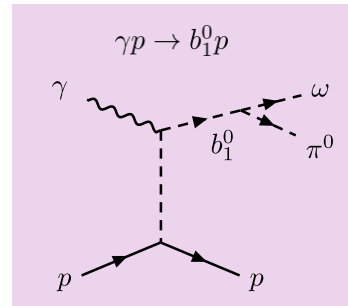
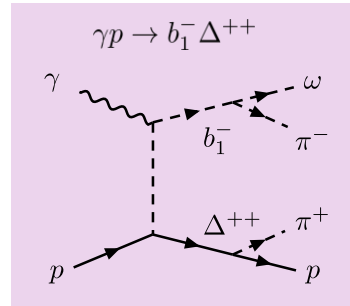
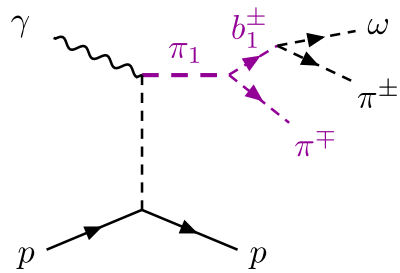
- ✓ More statistics
- ✗ Complicated final state



- ✗ Less statistics
- ✓ Easier (less particles)

Search for exotic hybrid mesons in photoproduction at JLab

- Amplitude analyses of multi-meson final states require models for production amplitudes of several processes.
- Collaboration between experimentalists and theorists.

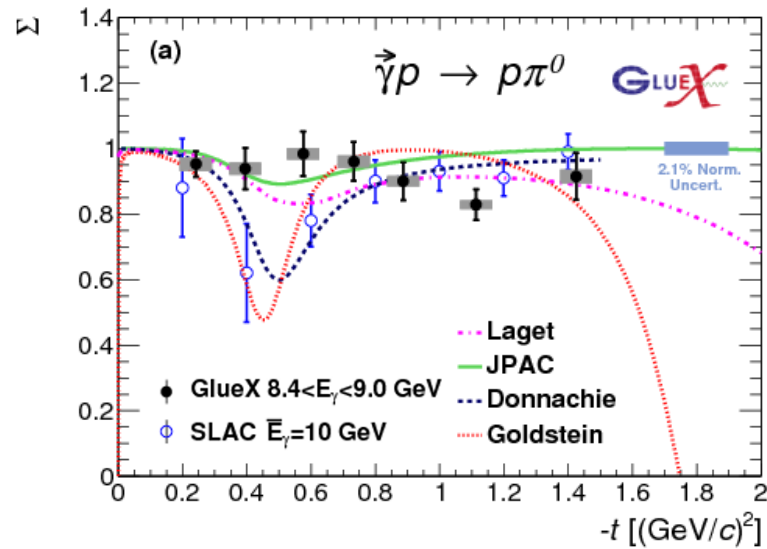


- Begin by understanding non-exotic J^{PC} production mechanism.

Production mechanism

- Neutral exchange reactions:

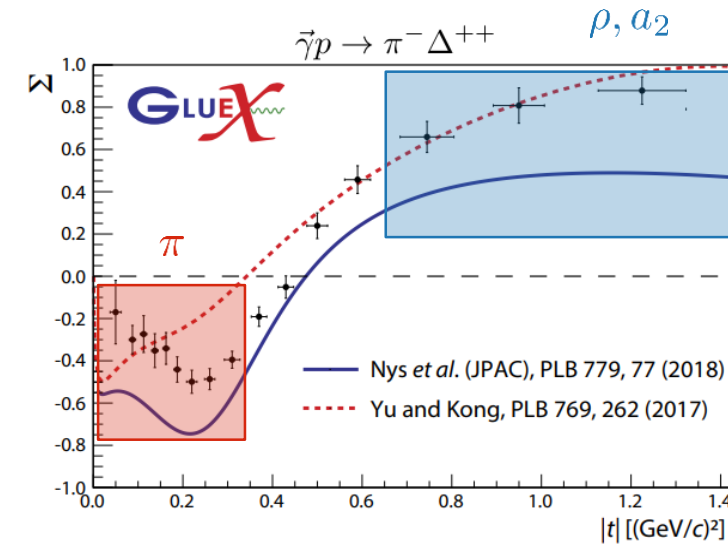
- Natural parity exchanges dominate



[GlueX Collaboration, *Phys.Rev.C* 95 (2017) 4, 042201]

- Charge exchange reactions:

- Small $-t$: unnatural exchanges favored
- Large $-t$: natural exchanges favored



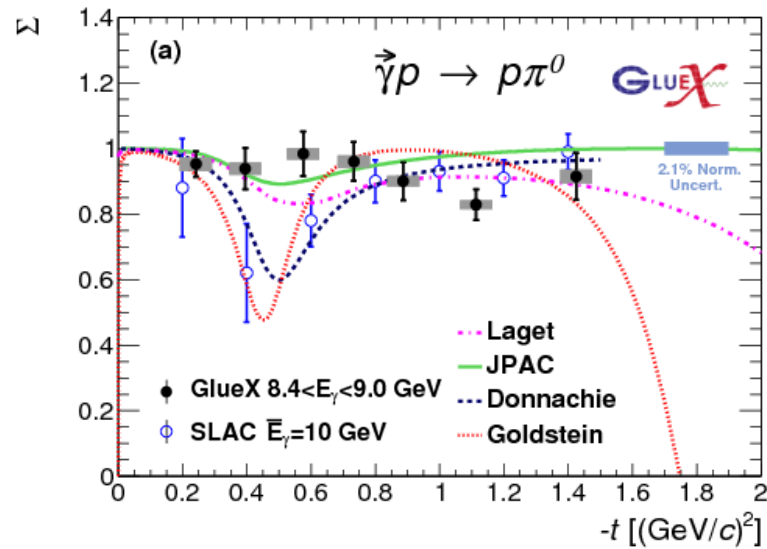
[GlueX Collaboration, *Phys.Rev.C* 103 (2021) 2, L022201]

- Crucial in the light (e.g. hybrid meson searches) and heavy (e.g. XYZ phenomenology) sectors.

Production mechanism

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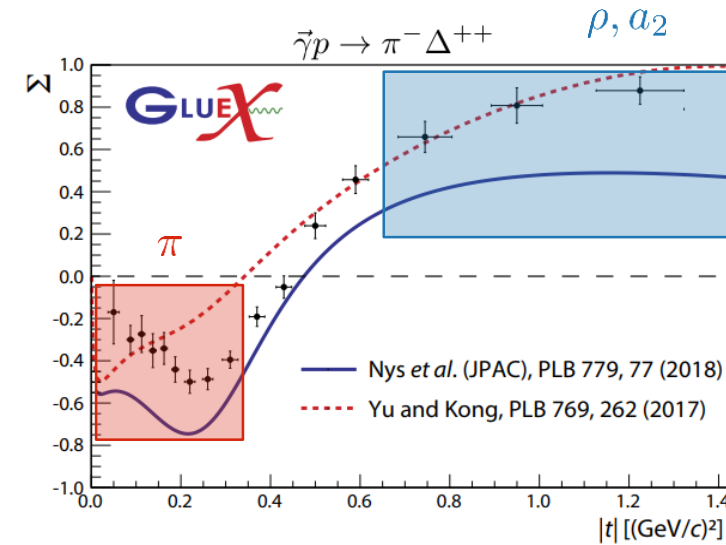
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Focus of this talk: Pion exchange mechanism

OUTLINE

- Motivation
- Remarks on Scattering and Regge Theory
- Pion exchange in pion photoproduction
 - Role of gauge invariance
 - Reggeization
- Other photoproduction reactions
 - $\eta^{(\prime)}\pi$
 - $b_1(1235)$
 - $\Delta^{++}(1232)$
- Conclusions

Principles of Scattering Theory

Unitarity $SS^\dagger = 1$

if $S = 1 + iA$ then $-i(A - A^\dagger) = 2\text{Im} A = AA^\dagger$

Analyticity

$A(s)$ has singularities (poles and branch cuts) in the complex s plane.

Crossing symmetry $A_{ab \rightarrow cd}(s, t, u) = A_{a\bar{c} \rightarrow \bar{b}d}(t, s, u)$

Physical regions (in the case of equal mass particles):

s -channel: $a + b \rightarrow c + d$ $s \geq 4m^2, t \leq 0, u \leq 0$

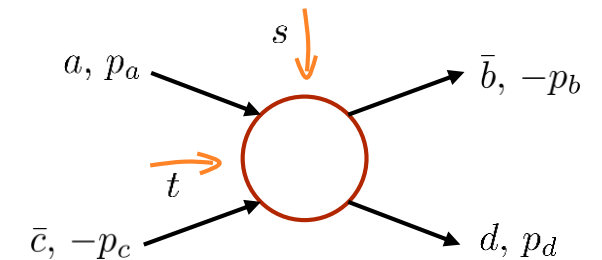
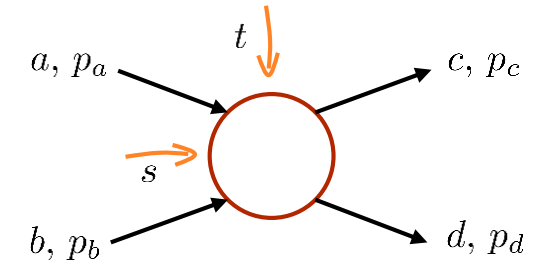
t -channel: $a + \bar{c} \rightarrow \bar{b} + d$ $t \geq 4m^2, s \leq 0, u \leq 0$

Partial wave expansion (no spin)

$$A(s, t) = \sum_{\ell=0}^{\infty} (2\ell + 1) f_{\ell}(s) P_{\ell}(z)$$

$f_{\ell}(s)$: s -channel partial wave amplitudes

$P_{\ell}(z)$: Legendre polynomials



Duality

[Dolen, Horn, Schmid (1968), Veneziano (1968)]

- Property of scattering amplitude.
- Connection between low-energy and high-energy domains.

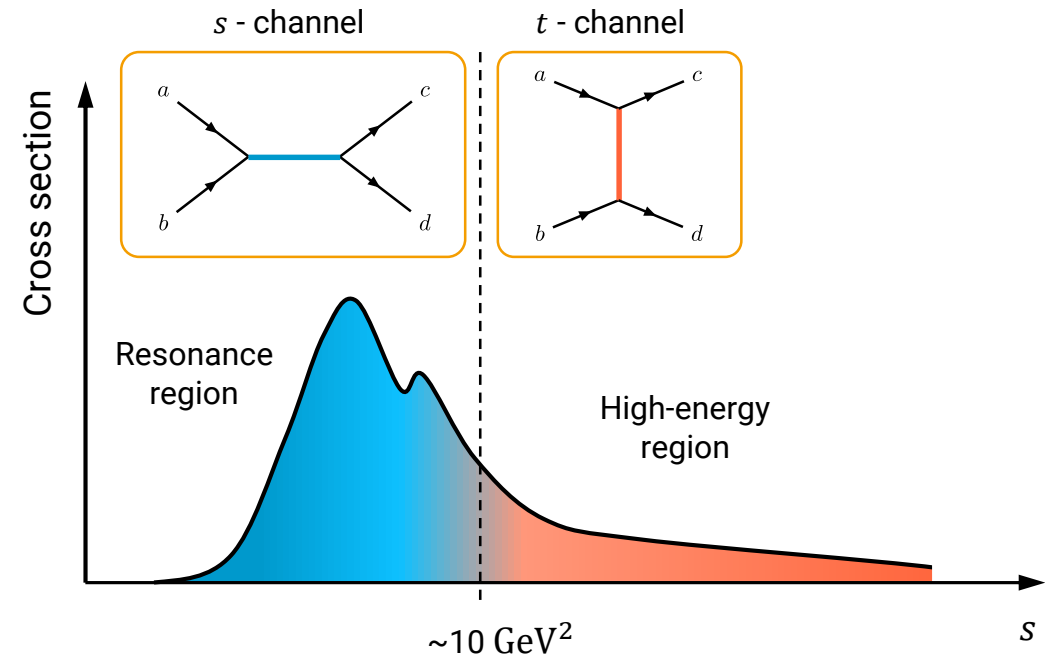
Low-energy: resonances

$$A(s, t) \sim \sum_r \frac{g_r}{s - s_r}$$

High energy: Regge exchanges

$$A(s, t) \sim \sum_i \frac{g_i}{t - t_i}$$

Analytically
connected



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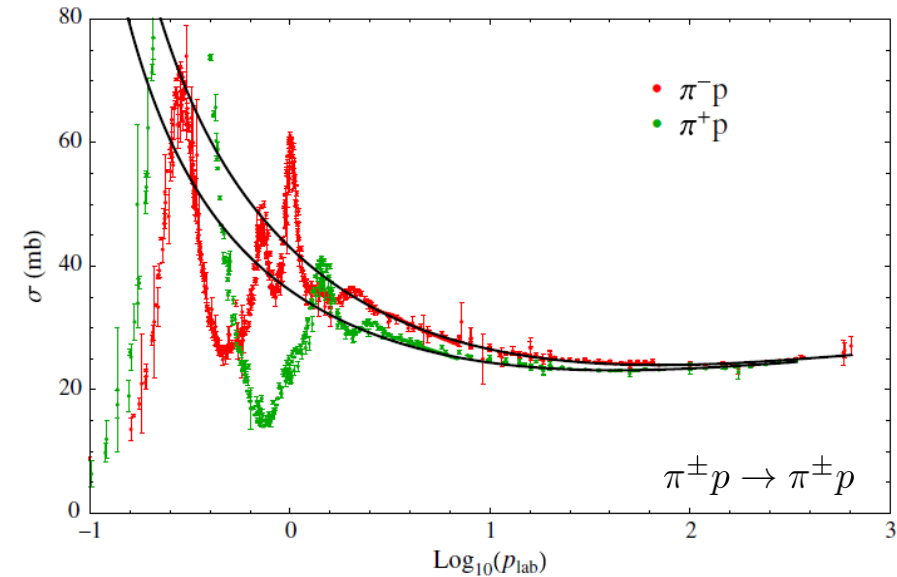
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Analytically
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[V.Mathieu et al., *Phys.Rev.D* 92 (2015) 7]

Regge Theory

- At high energies, the scattering amplitudes in the physical region of the s -channel are related to particle exchanges in the t -channel.

$$A(s, t) = \sum_{\ell=0}^{\infty} (2\ell + 1) f_{\ell}(t) P_{\ell}(z_t)$$

Regge limit
 $s \gg -t, m^2$

$$A(s, t) \sim s^{\ell_{\text{eff}}}$$

t -channel partial wave amplitudes

$$z_t = \cos \theta_t = 1 + \frac{2s}{t - 4m^2}$$

$$\lim_{z \rightarrow \infty} P_{\ell}(z) \sim z^{\ell}$$

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t -channel partial wave amplitudes $z_t = \cos \theta_t = 1 + \frac{2s}{t - 4m^2}$
 $\lim_{z \rightarrow \infty} P_{\ell}(z) \sim z^{\ell}$

- The concept of partial wave can be extended to complex values of angular momentum.

$\{f_{\ell}(t)\} \rightarrow f(\ell, t)$ with $f(\ell, t) \rightarrow f_{\ell}(t), \ell \in \{0, 1, 2, \dots\}$

Even and odd angular momenta have to be continued separately.

$$f_{\ell}(t) = \frac{1}{2} \int_{-1}^{+1} dz_t P_{\ell}(z_t) A(s, t)$$

$$A(s, t) = \frac{1}{\pi} \left[\int_{z_0}^{\infty} dz \frac{D_s(z, t)}{z - z_t} + \int_{z_0}^{\infty} dz \frac{D_u(-z_u, t)}{z_u + z_t} \right]$$

}

Froissart-Gribov projection

$$f_{\ell}^{\pm}(t) = \frac{1}{\pi} \int_{z_0}^{\infty} dz \{D_s(z, t) \pm D_u(-z, t)\} Q_{\ell}(z)$$

}

$f_{\ell}(t) = f^{+}(\ell, t)$
for even ℓ

$f_{\ell}(t) = f^{-}(\ell, t)$
for odd ℓ

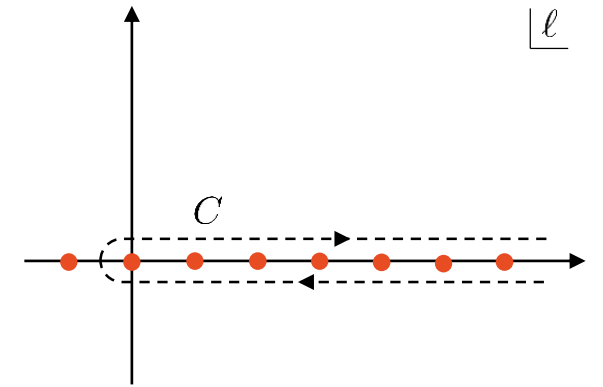
Regge Theory

$$A(s, t) = A^+(s, t) + A^-(s, t) \quad \text{with} \quad A^\pm(s, t) = \sum_{\ell=0}^{\infty} (2\ell + 1) f_\ell^\pm(t) \frac{1}{2} (P_\ell(z_t) \pm P_\ell(-z_t))$$

signature ↙

Procedure: Sommerfeld-Watson transform.

$$A^\pm(s, t) = -\frac{1}{2i} \int_C \frac{(2\ell + 1) f_\ell^\pm(\ell, t)}{\sin \pi \ell} \frac{1}{2} (P_\ell(-z_t) \pm P_\ell(z_t)) d\ell$$



Regge Theory

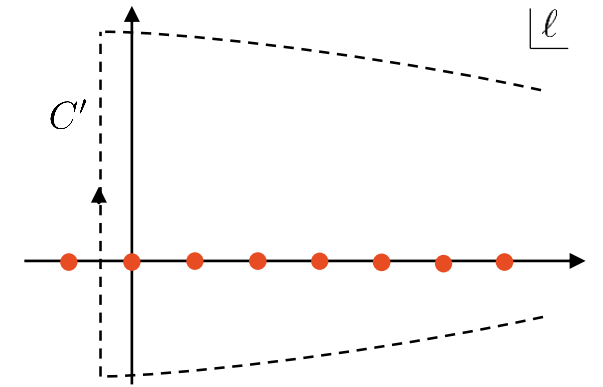
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The next step is to deform the contour.



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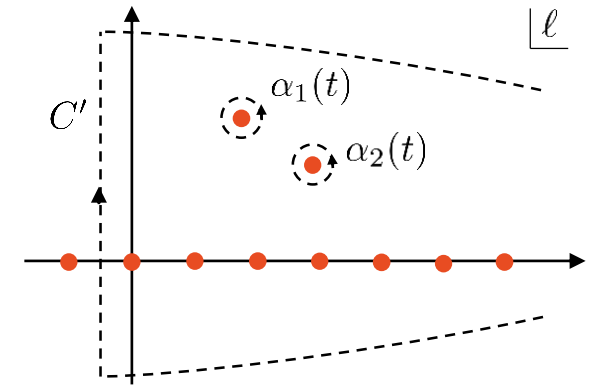
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The next step is to deform the contour.

We consider that the only singularities of $f^\pm(\ell, t)$ in the region $\ell > -\frac{1}{2}$ are poles in the upper half ℓ plane.



$$A^\pm(s, t) = -\frac{1}{2i} \int_{-\frac{1}{2}-i\infty}^{-\frac{1}{2}+i\infty} d\ell \dots - \sum_i \underbrace{\frac{\pi(2\alpha_i^\pm(t) + 1)\beta_i^\pm(t)}{\sin(\pi\alpha_i^\pm(t))} \frac{1}{2} [P_{\alpha_i^\pm}(-z_t) \pm P_{\alpha_i^\pm}(z_t)]}_{\text{Contribution from each pole} \sim s^{\alpha(t)}}$$

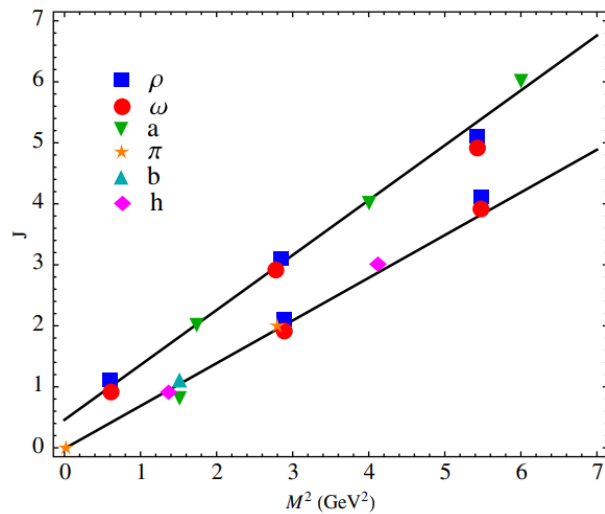
background $\sim s^{-1/2}$

$$A(s, t) \underset{s \rightarrow \infty}{\approx} \sum_i \left[\frac{\pi(2\alpha_i^+(t) + 1)\beta_i^+(t)}{\sin(\pi\alpha_i^+(t))} \frac{(1 + e^{-i\pi\alpha_i^+(t)})}{2} \left(\frac{s}{s_0}\right)^{\alpha_i^+(t)} - \frac{\pi(2\alpha_i^-(t) + 1)\beta_i^-(t)}{\sin(\pi\alpha_i^-(t))} \frac{(1 - e^{-i\pi\alpha_i^-(t)})}{2} \left(\frac{s}{s_0}\right)^{\alpha_i^-(t)} \right]$$

Reggeon trajectories

- Families with same quantum numbers but different spin J (even or odd).
- Almost straight lines (Chew-Frautschi plot)
- In standard Regge theory parameterized by:

$$\alpha(t) = \alpha' t + \alpha_0$$



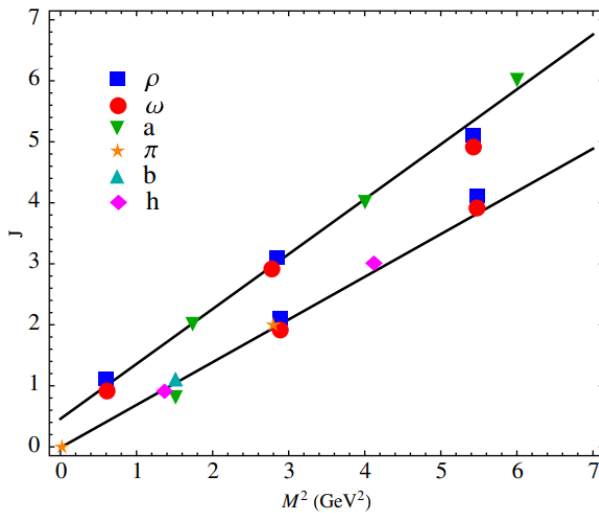
[V.Mathieu et al., *Phys.Rev.D* 98 (2018) 1, 014041]

$$\alpha_\pi(t) = 0.7(t - m_\pi^2) = 0.7t - 0.014$$

$$\alpha_\rho(t) = 0.9(t - m_\rho^2) + 1 = 0.9t + 0.466$$

Reggeon trajectories

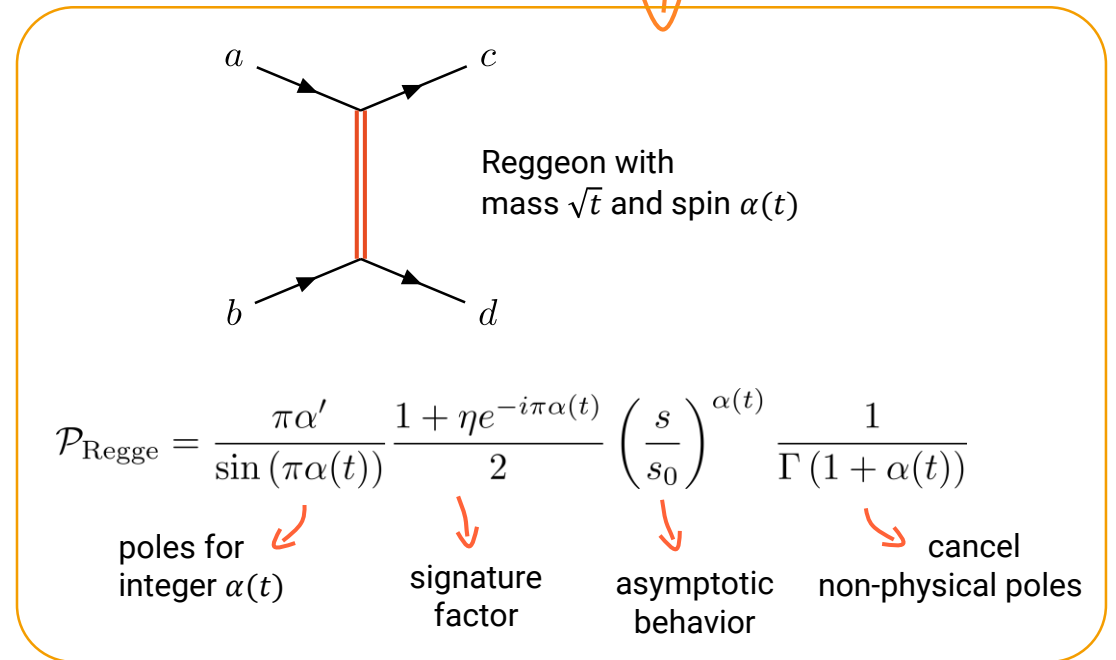
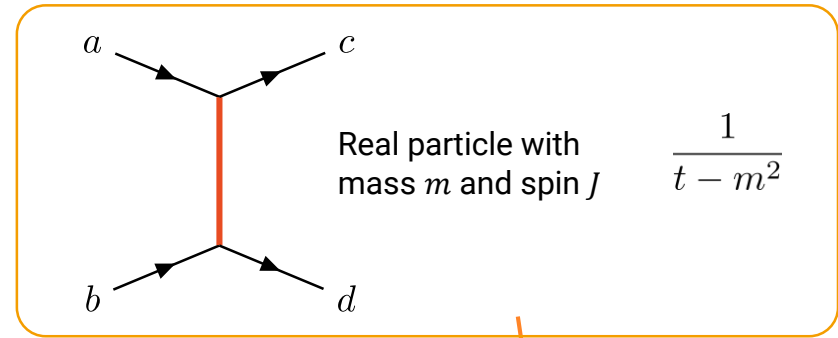
- Families with same quantum numbers but different spin J (even or odd).
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$$\alpha_\pi(t) = 0.7(t - m_\pi^2) = 0.7t - 0.014$$

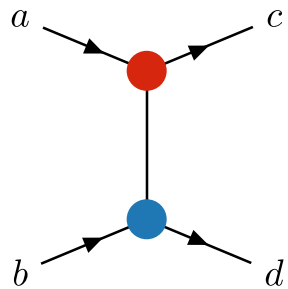
$$\alpha_\rho(t) = 0.9(t - m_\rho^2) + 1 = 0.9t + 0.466$$



Implications of Regge pole amplitudes

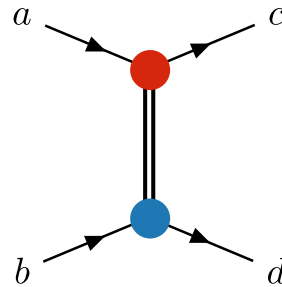
Factorization

Amplitude for particle exchange “factorizes”
(follows from unitarity).



$$A(s, t) = g_{ac} \frac{1}{t - m^2} g_{bd}$$

The reggeon residue $\beta(t)$:



- Contains all information about incoming and outgoing particles.
- Related to the reggeon-hadron interaction vertices.
- Satisfies factorization: $\beta(t) = \beta_{ac}(t)\beta_{bd}(t)$

Power law energy dependence

$$A(s, t) \sim s^{\alpha(t)}$$

$$\frac{d\sigma}{dt} \sim \frac{1}{s^2} |A(s, t)|^2 = s^{2-2\alpha(t)}$$

Leading Regge poles (biggest $\alpha(t)$) dominate asymptotically.

Phase

The phase comes from the signature factor: $\frac{1 + \eta e^{-i\pi\alpha(t)}}{2}$

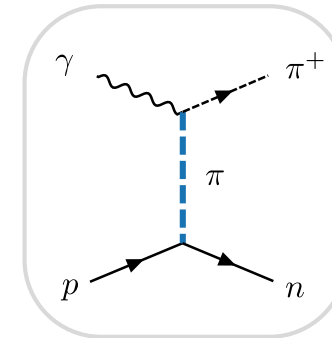
Exchange degeneracy (equal trajectories with opposite signatures) leads to rotating or constant phases.

- Corrections to these hypothesis, usually $\sim 10-20\%$. [J.Nys et al. (JPAC), *Phys.Rev.D* 98 (2018) 3, 034020]

Charged pion photoproduction

What do we know?

- Pion exchange dominates at small momentum transfer.
- Low energies: Constrained by effective Lagrangians of QCD.
- High energies: Regge theory.



$$A \sim \frac{t}{t - m_\pi^2}$$

$$A^R \sim (-t)^{(\mu_\gamma + |\mu_i - \mu_f|)/2}$$

Known issues

- Cannot describe forward cross-section data in $\gamma p \rightarrow \pi^+ n$ (same for $np \rightarrow pn$).
- What is pion exchange and how does it reggeize?

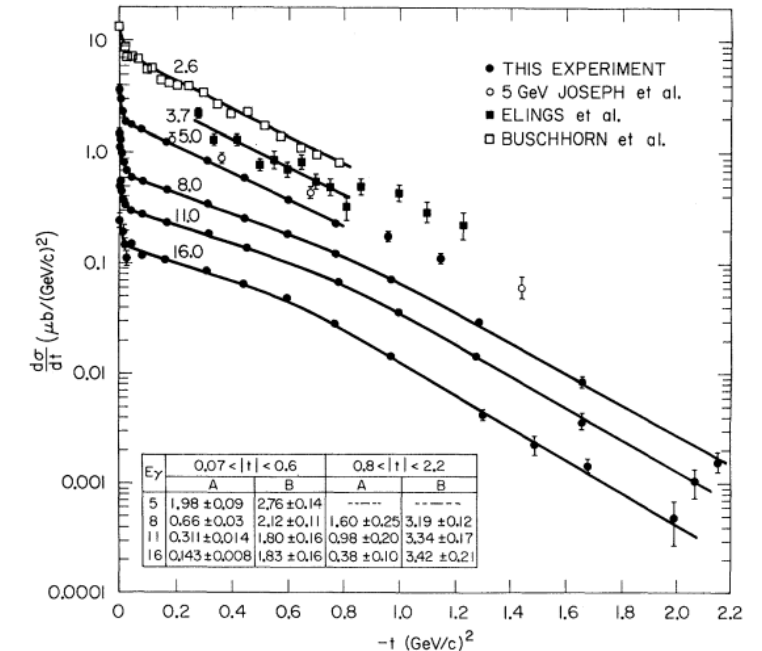
Proposed solutions

- Existence of parity-doublet conspirator of the pion.
- Regge cuts and absorption (final state interactions).
- Nucleon Born terms.

[J.S.Ball, W.R. Frazer and M. Jacob, *Phys.Rev.Lett.* 20 (1968) 518]

[F. Henyey, G.L.Kane, J.Pumplin, *Phys.Rev.* 182 (1969) 1579]

[L.Jones, *Rev.Mod.Phys.* 52 (1980) 545]

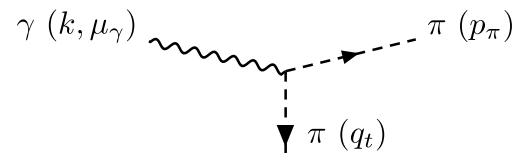


[A. Boyarski et al., *Phys.Rev.Lett.* 20 (1968) 300]

Adding the nucleon Born diagrams

- s-channel reaction: $\gamma(k, \mu_\gamma) + N(p_i, \mu_i) \rightarrow \pi(p_\pi) + N(p_f, \mu_f)$
- Helicity amplitude: $A_{\mu_\gamma \mu_i \mu_f} = \epsilon_{\mu_\gamma}(k) \cdot J_{\mu_i \mu_f}$

t - channel Born diagram



$$J_{\mu_i \mu_f, t}^\mu = -\sqrt{2} e_\pi g_{\pi NN} \frac{q_t^\mu - p_\pi^\mu}{t - \mu^2} \bar{u}_{\mu_f}(p_f) \gamma_5 u_{\mu_i}(p_i)$$

$$g_{\pi NN} = 13.48 \rightarrow \text{PS } \pi NN \text{ coupling}$$

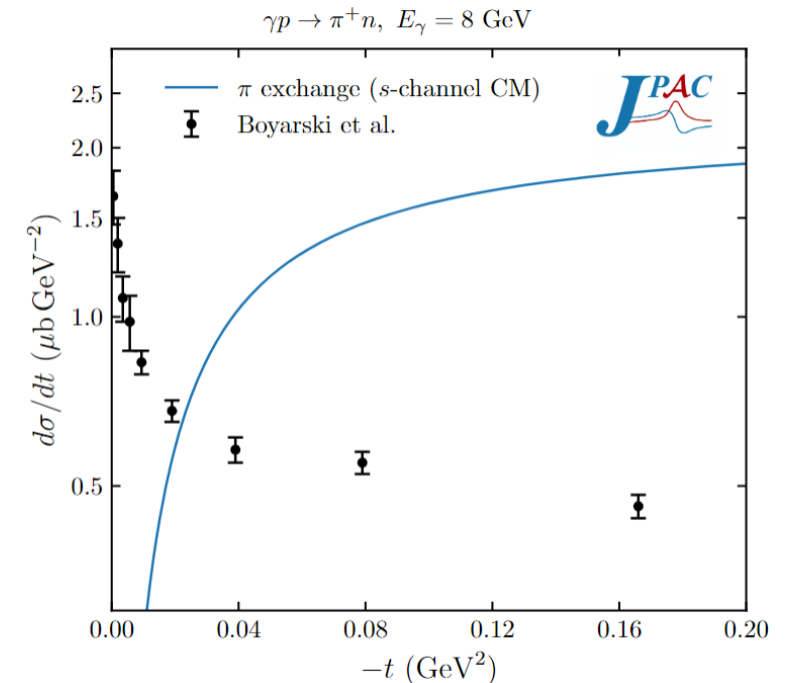


Current is not conserved



amplitude is not gauge invariant (frame dependent)

- Pion exchange cannot reproduce experimental cross section at small momentum transfer



[G.Montana et al. (in preparation)]

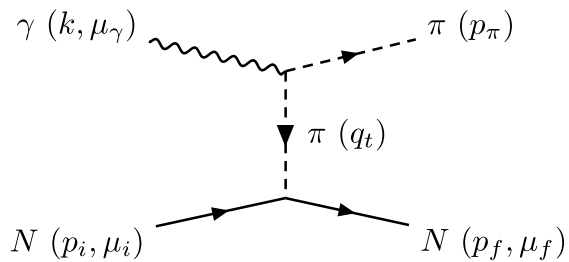
Adding the nucleon Born diagrams

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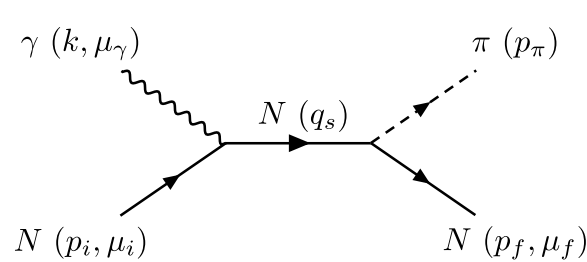
$$J_{\mu_i \mu_f}^\mu = J_{\mu_i \mu_f, t}^\mu + J_{\mu_i \mu_f, s}^\mu + J_{\mu_i \mu_f, u}^\mu \quad \longrightarrow \quad \text{Total current is conserved}$$

t - channel Born diagram



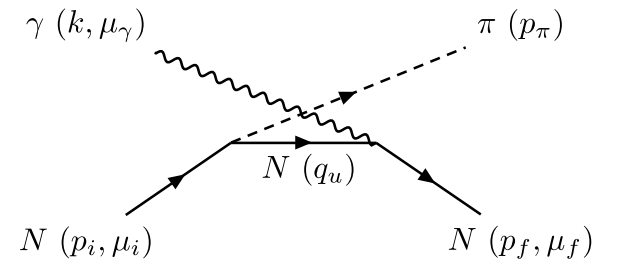
$$J_{\mu_i \mu_f, t}^\mu = -\sqrt{2}e_\pi g_{\pi NN} \frac{q_t^\mu - p_\pi^\mu}{t - \mu^2} \bar{u}_{\mu_f}(p_f) \gamma_5 u_{\mu_i}(p_i)$$

s-channel



$$J_{\mu_i \mu_f, s}^\mu = \sqrt{2}e_N g_{\pi NN} \bar{u}_{\mu_f}(p_f) \gamma_5 \frac{\not{q}_s + M}{s - M^2} \gamma^\mu u_{\mu_i}(p_i)$$

u-channel



$$J_{\mu_i \mu_f, u}^\mu = \sqrt{2}e_N g_{\pi NN} \bar{u}_{\mu_f}(p_f) \gamma^\mu \frac{\not{q}_u + M}{u - M^2} \gamma_5 u_{\mu_i}(p_i)$$

• Separate electric and magnetic contributions: $A_{\mu_\gamma \mu_i \mu_f} = A_{\mu_\gamma \mu_i \mu_f}^e + A_{\mu_\gamma \mu_i \mu_f}^m$

$$A_{\mu_\gamma \mu_i \mu_f}^e = 2\sqrt{2}g_{\pi NN} \left[e_\pi \frac{(\epsilon_{\mu_\gamma} \cdot p_\pi)}{t - \mu^2} + e_{N_i} \frac{(\epsilon_{\mu_\gamma} \cdot p_i)}{s - M^2} + e_{N_f} \frac{(\epsilon_{\mu_\gamma} \cdot p_f)}{u - M^2} \right] \bar{u}_{\mu_f}(p_f) \gamma_5 u_{\mu_i}(p_i)$$

$$A_{\mu_\gamma \mu_i \mu_f}^m = \sqrt{2}g_{\pi NN} \left[\frac{e_{N_i}}{s - M^2} + \frac{e_{N_f}}{u - M^2} \right] \bar{u}_{\mu_f}(p_f) \gamma_5 \not{k}_f \not{\epsilon}_{\mu_\gamma} u_{\mu_i}(p_i)$$

Electric term

$$A_{\mu_\gamma \mu_i \mu_f}^e = 2\sqrt{2}g_{\pi NN} \left[e_\pi \frac{(\epsilon_{\mu_\gamma} \cdot p_\pi)}{t - \mu^2} + e_{N_i} \frac{(\epsilon_{\mu_\gamma} \cdot p_i)}{s - M^2} + e_{N_f} \frac{(\epsilon_{\mu_\gamma} \cdot p_f)}{u - M^2} \right] \bar{u}_{\mu_f}(p_f) \gamma_5 u_{\mu_i}(p_i)$$

- Using momentum conservation and electric charge conservation ($e_{N_i} = e_\pi - e_{N_f}$):

$$A_{\mu_\gamma \mu_i \mu_f}^e = 2\sqrt{2}g_{\pi NN} \left[e_\pi \left(\frac{(\epsilon_{\mu_\gamma} \cdot p_\pi)}{t - \mu^2} + \frac{(\epsilon_{\mu_\gamma} \cdot (p_i + p_f))}{s - u} \right) + \frac{1}{2} e_{N_i} \left(\frac{(\epsilon_{\mu_\gamma} \cdot p_\pi)}{s - M^2} + \frac{(\epsilon_{\mu_\gamma} \cdot (p_i + p_f))}{s - u} \frac{t - \mu^2}{s - M^2} \right) - \frac{1}{2} e_{N_f} \left(\frac{(\epsilon_{\mu_\gamma} \cdot p_\pi)}{u - M^2} + \frac{(\epsilon_{\mu_\gamma} \cdot (p_i + p_f))}{s - u} \frac{t - \mu^2}{u - M^2} \right) \right] \bar{u}_{\mu_f}(p_f) \gamma_5 u_{\mu_i}(p_i)$$

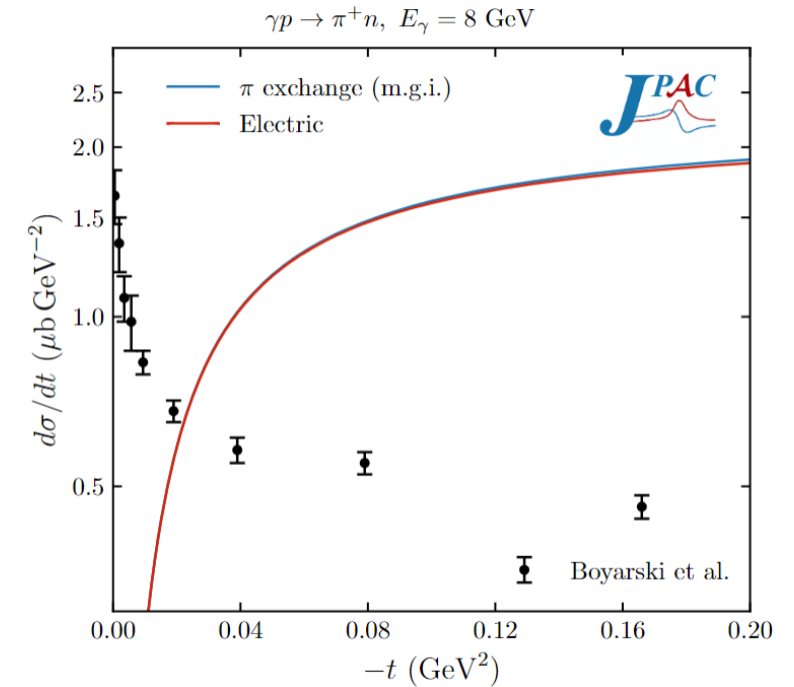
Minimal gauge invariant (m.g.i.)

- Differential cross section

$$\left(\frac{d\sigma}{dt} \right)_{\pi\text{-m.g.i.}} = 4 \left(\frac{s - M^2}{s - u} \right)^2 \left(\frac{d\sigma}{dt} \right)_{\pi\text{-bare, CM}} \stackrel{t \rightarrow t_{\min}}{\approx} \left(\frac{d\sigma}{dt} \right)_{\pi\text{-bare, CM}}$$

$$\left(\frac{d\sigma}{dt} \right)_{e, \gamma p \rightarrow \pi^+ n} = \left(\frac{d\sigma}{dt} \right)_{\pi\text{-bare, CM}}$$

$$\left(\frac{d\sigma}{dt} \right)_{e, \gamma n \rightarrow \pi^- p} = 4 \left(\frac{s - M^2}{M^2 - u} \right)^2 \left(\frac{d\sigma}{dt} \right)_{\pi\text{-bare, CM}} \stackrel{t \rightarrow t_{\min}}{\approx} \left(\frac{d\sigma}{dt} \right)_{\pi\text{-bare, CM}}$$



[G.Montana et al. (in preparation)]

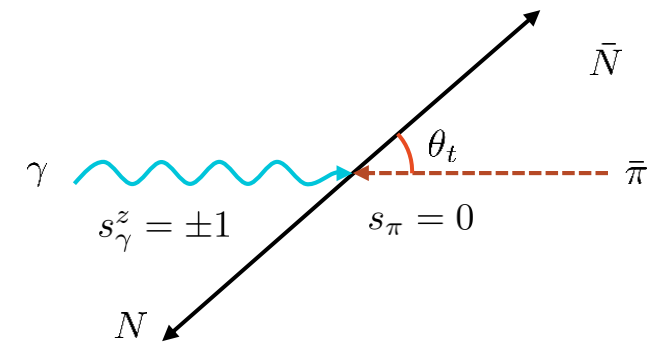
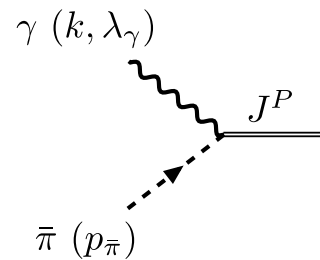
Pion pole in the t -channel: where does it come from?

- t -channel reaction: $\gamma(k, \lambda_\gamma) + \bar{\pi}(-p_\pi) \rightarrow \bar{N}(-p_i, \lambda_i) + N(p_f, \lambda_f)$
- t -channel partial wave expansion $A(s, t) = \sum_{J=0}^{\infty} (2J+1) A_J(t) d_{\lambda_\gamma, \lambda_i - \lambda_f}^J(z_t)$

Partial wave with $J^P = 0^-$ has the singularity closest to the physical region

... but π contribution to t -channel vanishes!

↪ M_π would have to be ± 1



- Crossing symmetry implies (parity conserving) helicity amplitudes in s - and t -channels are related by a rotation.
- The nucleon Born terms contain a “pion pole” that arises from kinematical factors.

$$A_{\lambda_\gamma \lambda_i \lambda_f}^e = 2\sqrt{2}g_{\pi NN} \left(e_\pi + \frac{1}{2}e_{N_i} \frac{t - \mu^2}{s - M^2} - \frac{1}{2}e_{N_f} \frac{t - \mu^2}{u - M^2} \right) \frac{1}{s - u} (\epsilon_{\lambda_\gamma} \cdot (p_i + p_f)) \bar{u}_{\lambda_f}(p_f) \gamma_5 v_{\lambda_i}(-p_i)$$

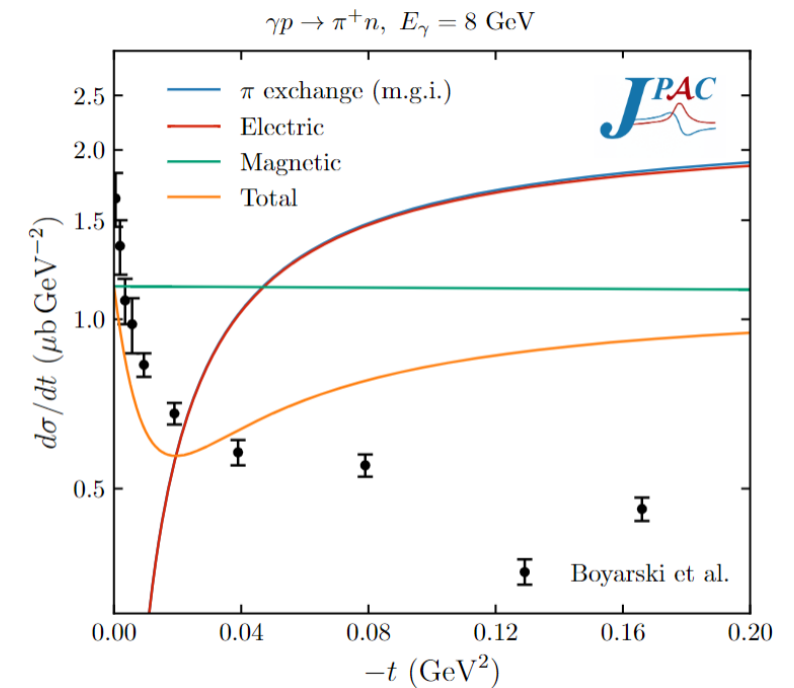
$$\approx i2g_{\pi NN} \lambda_\gamma 2\lambda_i \delta_{\lambda_i \lambda_f} \left(e_\pi + \frac{1}{2}e_{N_i} \frac{t - \mu^2}{s - M^2} - \frac{1}{2}e_{N_f} \frac{t - \mu^2}{u - M^2} \right) \frac{t}{t - \mu^2}$$

Magnetic term

$$A_{\mu_\gamma \mu_i \mu_f}^m = \sqrt{2} g_{\pi NN} \left[\frac{e_{N_i}}{s - M^2} + \frac{e_{N_f}}{u - M^2} \right] \bar{u}_{\mu_f}(p_f) \gamma_5 \not{k} \not{\epsilon}_{\mu_\gamma} u_{\mu_i}(p_i)$$

$$\approx \mu_\gamma 2g_{\pi NN} (e_{N_i} - e_{N_f}) \delta_{\mu_\gamma \mu_i} \delta_{-\mu_i \mu_f}$$

- At $t \sim 0$ the electric term of the amplitude vanishes.
- The magnetic term has small dependence in t .
- Size of the cross section agrees with the data at $t \sim 0$.
- No need for alternative (unphysical) explanations of the experimental data:
 - Over-absorption
 - Parity doublet conspirator



[G.Montana et al. (in preparation)]

Reggeization of pion exchange

- The exchanged pion is expected to reggeized.
- In the Regge-pole approximation:

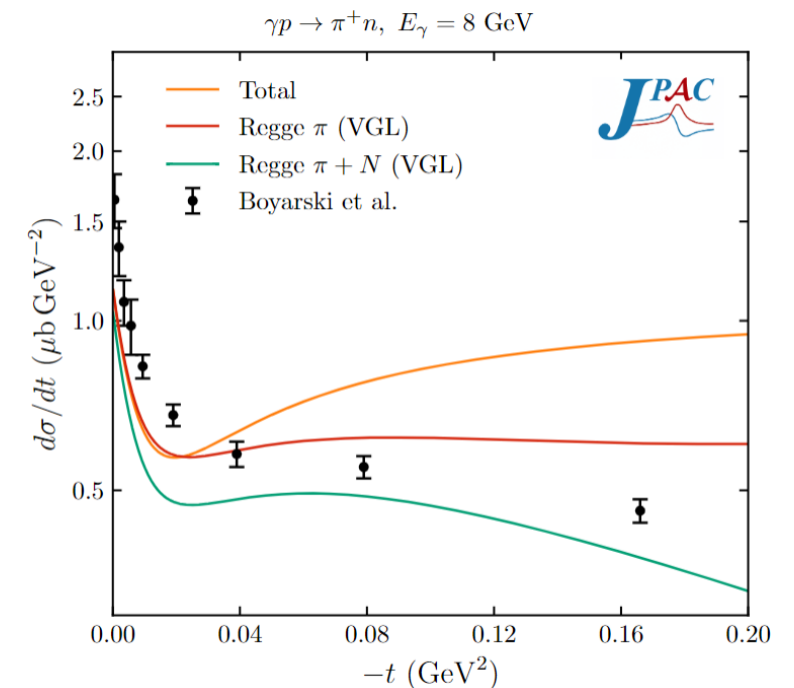
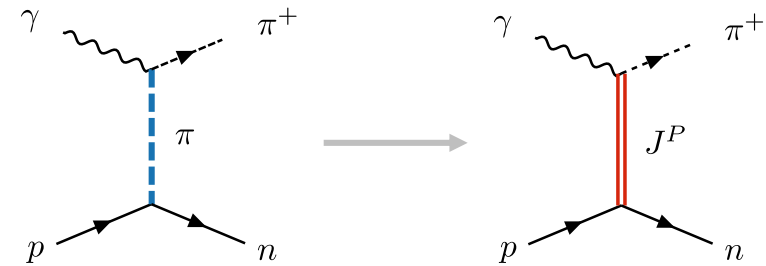
$$\frac{1}{t - \mu^2} \longrightarrow \mathcal{P}_\pi^{\text{Regge}} = \frac{\pi \alpha'_\pi}{2} \frac{1 + e^{-i\pi \alpha_\pi(t)}}{\sin \pi \alpha_\pi(t)} \left(\frac{s}{s_0} \right)^{\alpha_\pi(t)}$$

Pion trajectory: $\alpha_\pi(t) = \alpha'_\pi(t - \mu^2)$ with $\alpha'_\pi = 0.7$

- VGL model: reggeize full Born amplitude ($\pi + N$ exchanges, electric and magnetic).

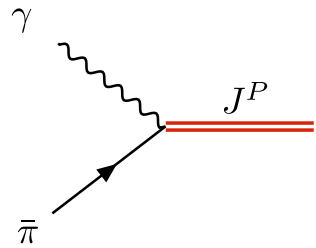
[M.Guidal, J.M.Laget and M. Vanderhaeghen, *Nucl.Phys.A* 627 (1997) 645-678]

→ What does it mean to reggeized the π exchange?



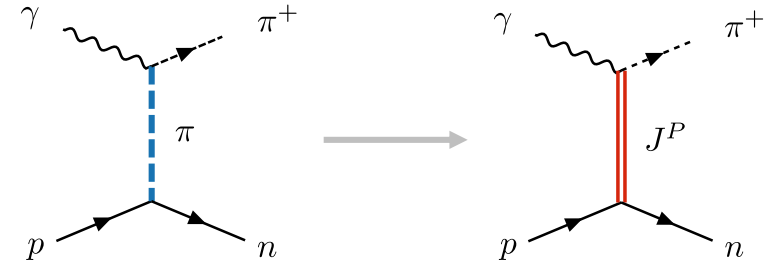
Reggeization of pion exchange

- Rigorous reggeization:
Explicit exchanges of t -channel partial waves in the π trajectory
- Vertices coupling $\gamma\pi$ and $N\bar{N}$ to $J^P = (\text{even})^-$:



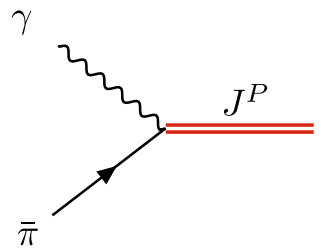
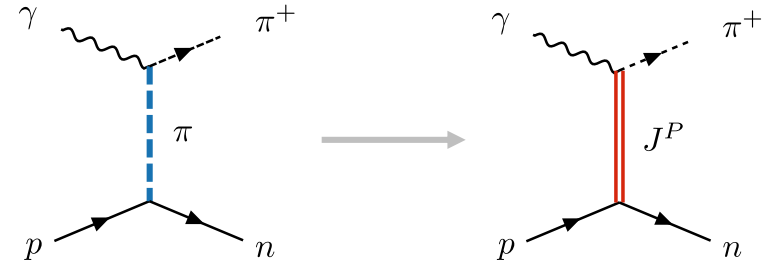
$$1^- \otimes 0^- = 1^+ \left\{ \begin{array}{l} L = 1 \\ L = \{J - 1, J + 1\} \end{array} \right. \left. \begin{array}{l} J = 0 \\ J \geq 2 \end{array} \right\} \text{one } L \text{ vs two } L\text{'s}$$

$$V_{\lambda_\gamma}(J) = 2\sqrt{2}e_{\bar{\pi}} \left[k^{\nu_1} \dots k^{\nu_J} \epsilon_\mu(k, \lambda_\gamma) p_{\bar{\pi}}^\mu - (k \cdot p_{\bar{\pi}}) k^{\nu_1} \dots k^{\nu_{J-1}} \epsilon^{\nu_J}(k, \lambda_\gamma) \right] \epsilon_{\nu_1, \dots, \nu_J}^*(M) \rightarrow \text{Gauge invariant by construction}$$



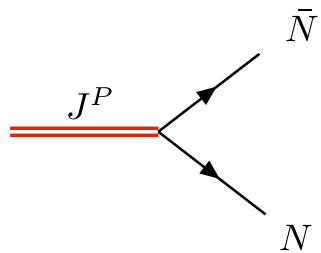
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$$\frac{1}{2}^+ \otimes \frac{1}{2}^- = 0^- \oplus 1^- \rightarrow L = J$$

↖ helicity non-flip ↖ helicity flip

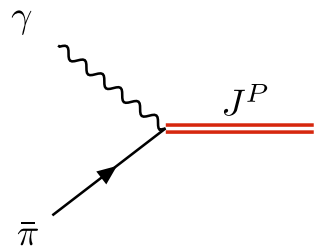
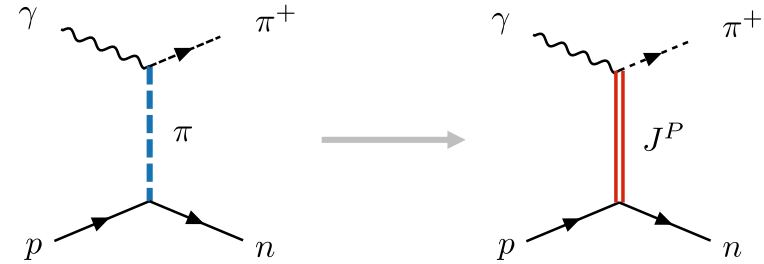
$$V_{\lambda_i \lambda_f}^{\text{NF}}(J) = g P^{\nu_1} \dots P^{\nu_J} \epsilon_{\nu_1, \dots, \nu_J}(M) \bar{u}_{\lambda_f}(p_f) \gamma_5 v_{\lambda_i}(p_i)$$

$$V_{\lambda_i \lambda_f}^{\text{F}}(J) = g P^{\nu_1} \dots P^{\nu_{J-1}} \epsilon_{\nu_1, \dots, \nu_J}(M) \bar{u}_{\lambda_f}(p_f) \gamma^{\nu_J} \gamma_5 v_{\lambda_i}(p_i)$$

$$P^\nu = p_i^\nu + p_f^\nu$$

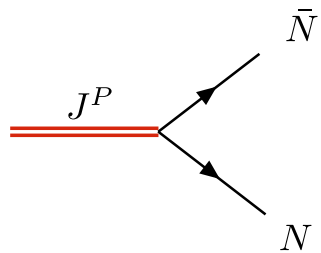
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$$\frac{1}{2}^+ \otimes \frac{1}{2}^- = 0^- \oplus 1^- \rightarrow L = J$$

helicity non-flip helicity flip

$$V_{\lambda_i \lambda_f}^{\text{NF}}(J) = g P^{\nu_1} \dots P^{\nu_J} \epsilon_{\nu_1, \dots, \nu_J}(M) \bar{u}_{\lambda_f}(p_f) \gamma_5 v_{\lambda_i}(p_i)$$

$$V_{\lambda_i \lambda_f}^{\text{F}}(J) = g P^{\nu_1} \dots P^{\nu_{J-1}} \epsilon_{\nu_1, \dots, \nu_J}(M) \bar{u}_{\lambda_f}(p_f) \gamma^{\nu_J} \gamma_5 v_{\lambda_i}(p_i)$$

$$P^\nu = p_i^\nu + p_f^\nu$$

$$V_{\lambda_\gamma}(J) \frac{1}{J - \alpha_\pi(t)} V_{\lambda_i \lambda_f}(J) = \frac{g_{\lambda_\gamma \lambda_i \lambda_f}^J(t)}{J - \alpha_\pi(t)} d_{\lambda_\gamma \lambda_i - \lambda_f}^J(\theta_t)$$

Regge pole propagator

partial wave amplitude Wigner d -function

Reggeization of pion exchange

- Analytical continuation to $J = 0$: $\alpha_\pi(t) = \alpha'_\pi(t - \mu^2)$

$$\left. \frac{g_{\lambda_\gamma \lambda_i \lambda_f}^J(t)}{J - \alpha_\pi(t)} d_{\lambda_\gamma \lambda_i - \lambda_f}^J(\theta_t) \right|_{J=0} \approx i2 \frac{g}{\alpha'_\pi} \lambda_\gamma 2\lambda_i \delta_{\lambda_i \lambda_f} e_\pi \frac{t}{t - \mu^2} \quad \text{with} \quad g = \alpha'_\pi g_{\pi NN}$$

↪ Recover m.g.i. π exchange (or electric term)

- Spin summation

(e.g. Sommerfeld-Watson transform, generating function of Jacobi polynomials)

$$A_{\lambda_\gamma \lambda_i \lambda_f}^R(s, t) = \sum_{J=0,2,\dots} (2J + 1) \frac{g_{\lambda_\gamma \lambda_i \lambda_f}^J(t)}{J - \alpha_\pi(t)} d_{\lambda_\gamma, \lambda_i - \lambda_f}^J(\theta_t)$$

$$g = \alpha'_\pi g_{\pi NN} \Lambda_J(r_t r_b)^J$$

microscopic structure



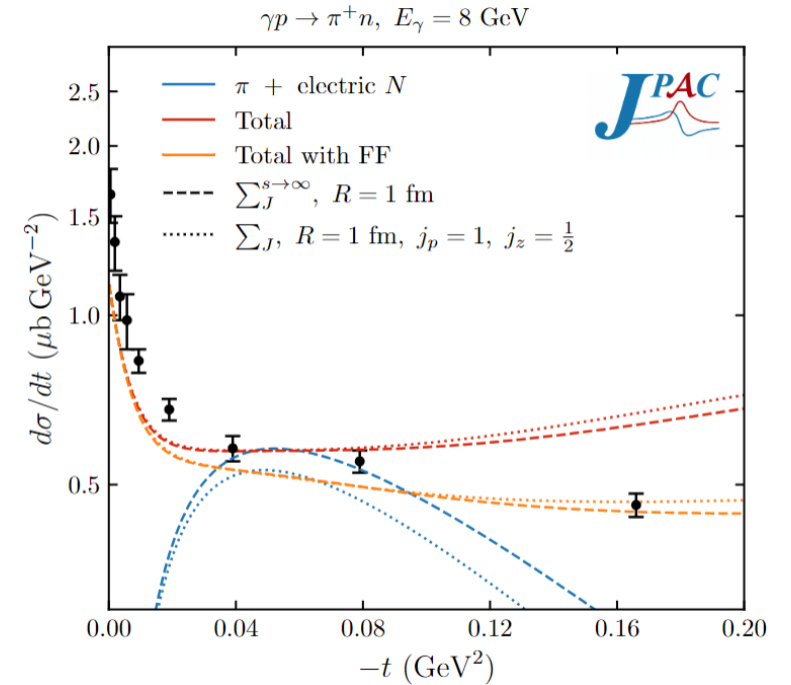
hadronic radii

$$c_J \Lambda_J(r_t r_b)^J \rightarrow \frac{j_p}{j_z} \frac{J + j_z}{J + j_p} R^{2J}$$



kinematic factors

- Corrections: add form factor to magnetic term $\beta(t) = \frac{\Lambda}{\Lambda^2 - t}$ with $\Lambda \sim 1 \text{ GeV}$



[G.Montana et al. (in preparation)]

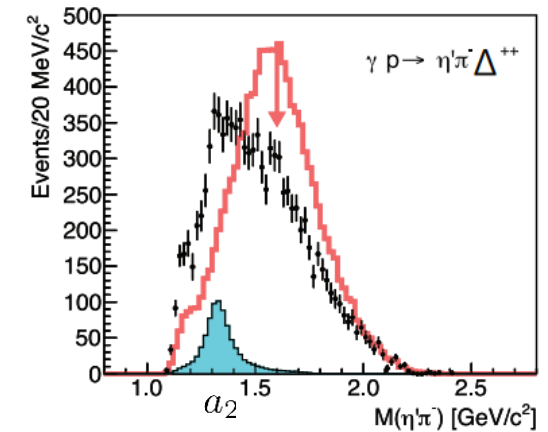
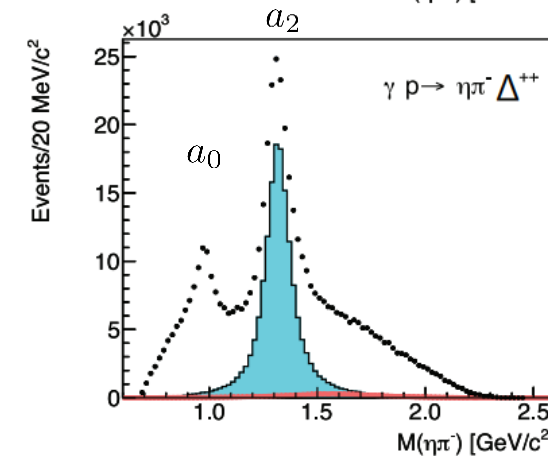
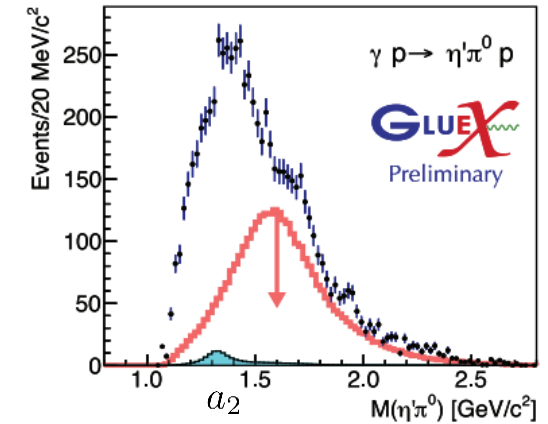
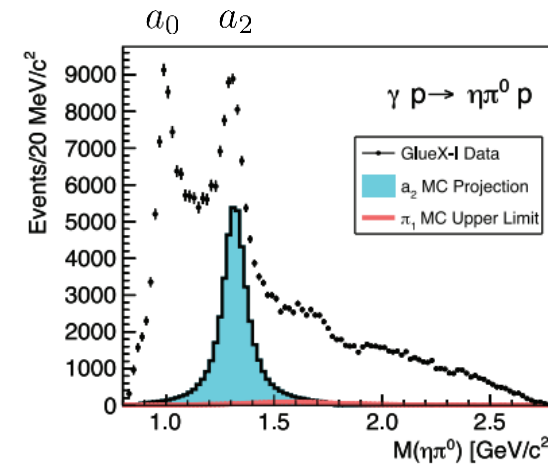
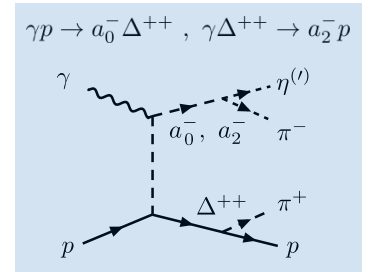
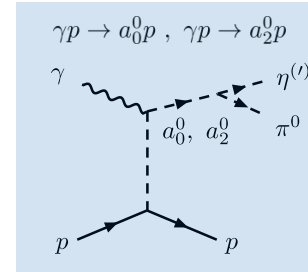
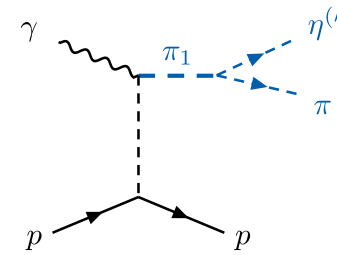
Photoproduction of $\eta^{(\prime)}\pi$

- GlueX can access different channels:

$$\gamma p \rightarrow \eta^{(\prime)}\pi^0 p$$

$$\gamma p \rightarrow \eta^{(\prime)}\pi^- \Delta^{++}$$

- Small signal of $\pi_1(1600)$ in partial wave with $\ell = 1$ (*P*-wave).
- Clear signals of non-exotic $a_0(980)$ (*S*-wave) and $a_2(1320)$ (*D*-wave).
- Overlapping resonances \rightarrow need partial-wave analysis.



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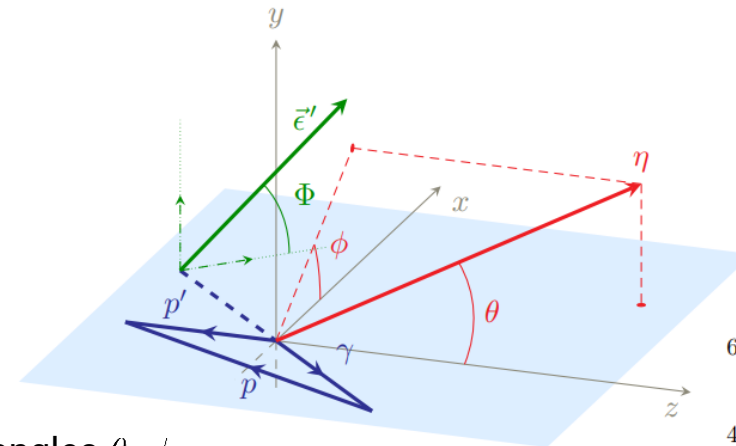
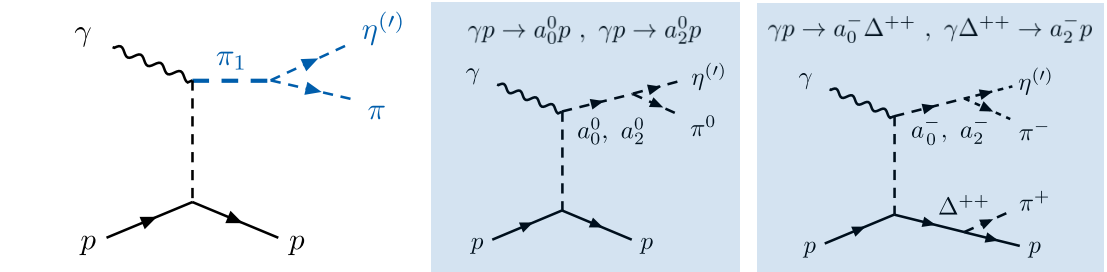
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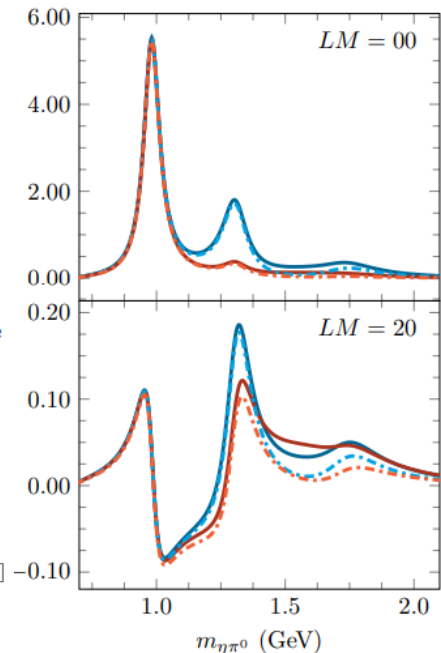
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- Three angles needed to describe the intensity: polarization Φ and decay angles θ, ϕ .
- Observables:

- Moments of angular distribution:
 - \rightarrow sensitive to exotic signal via interference.
- Polarization asymmetry
- Spin-density matrix elements (SDMEs)
 - \rightarrow 9 independent parameters

$$I(\Omega, \Phi) = \kappa \sum_{\lambda, \lambda', \lambda_1, \lambda_2} A_{\lambda; \lambda_1 \lambda_2}(\Omega) \rho_{\lambda \lambda'}^\gamma(\Phi) A_{\lambda'; \lambda_1 \lambda_2}^*(\Omega)$$



- $H^0(LM)$, full
- $H^1(LM)$, full
- - - $H^0(LM)$, no P-wave
- - - $H^1(LM)$, no P-wave



[V.Mathieu et al. (JPAC), Phys.Rev.D 100 (2019) 5, 054017]

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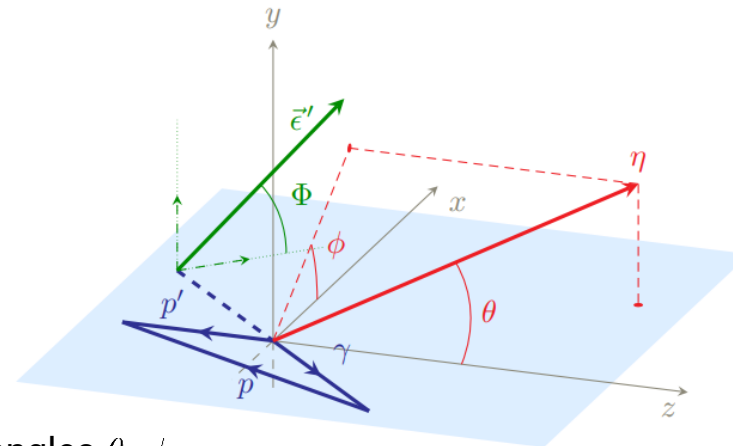
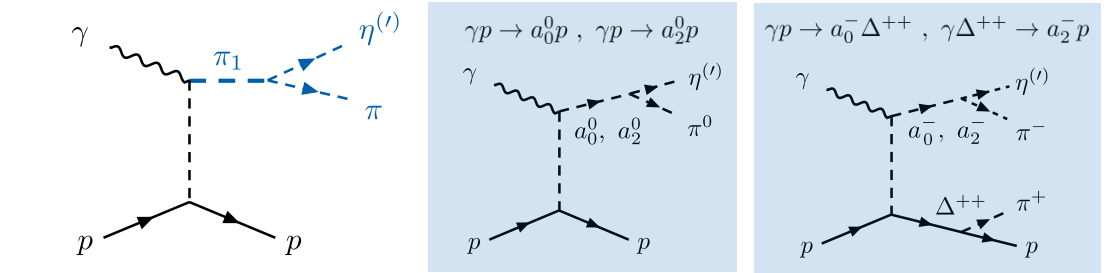
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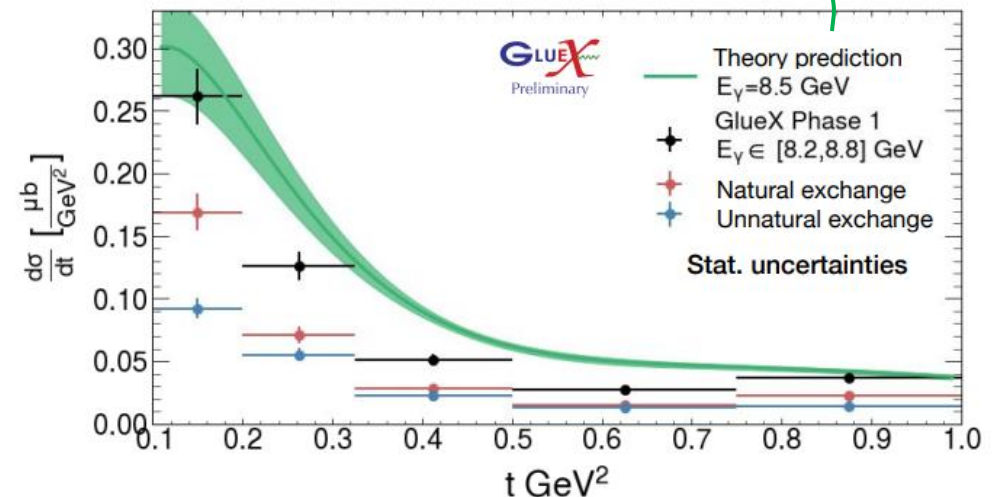
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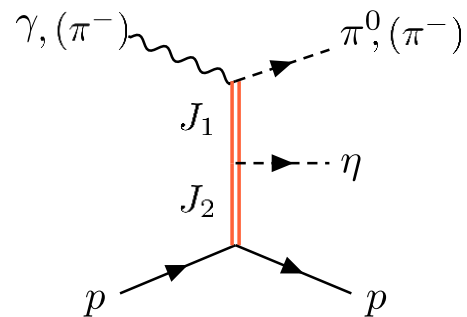
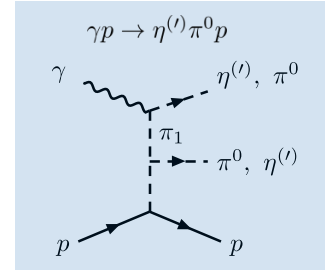
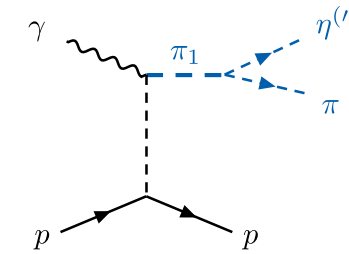


[V.Mathieu et al. (JPAC), Phys.Rev.D 102 (2020) 1, 014003]

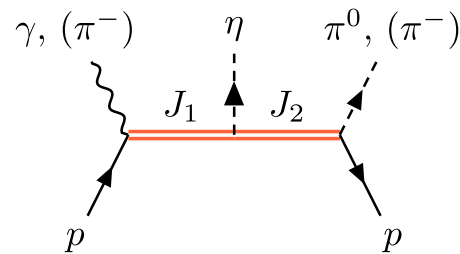


Double Regge contributions

- Contributes to background.
- Models from the 70's (e.g. Veneziano, Shimada) can't reproduce high-statistics data at COMPASS and GlueX.
- Need to develop new double-Regge amplitudes consistent with Regge phenomenology.



At high energy:



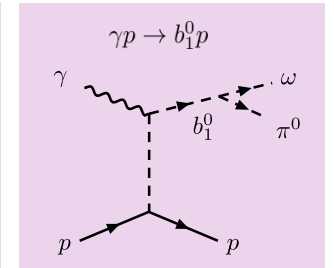
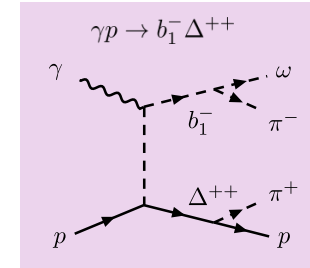
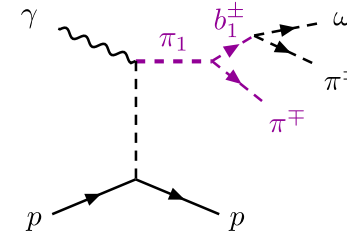
- Need to make the diagrams consistent with each other.
- Use quark string-breaking models to calculate helicity dependence explicitly, and give us insight into the Regge couplings.

Photoproduction of $b_1(1235)$

- Lattice QCD calculations predicts the dominant $\pi_1(1600)$ decay channel be the $b_1\pi(\rightarrow 5\pi)$.
- First step is to understand the b_1 production and decay to $\omega\pi$.
- GlueX can access charged and neutral b_1 :

$$\gamma p \rightarrow b_1^0 p \rightarrow \omega \pi^0 p$$

$$\gamma p \rightarrow b_1^- \Delta^{++} \rightarrow \omega \pi^- \Delta^{++}$$



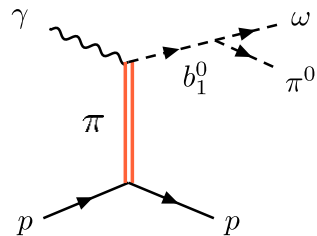
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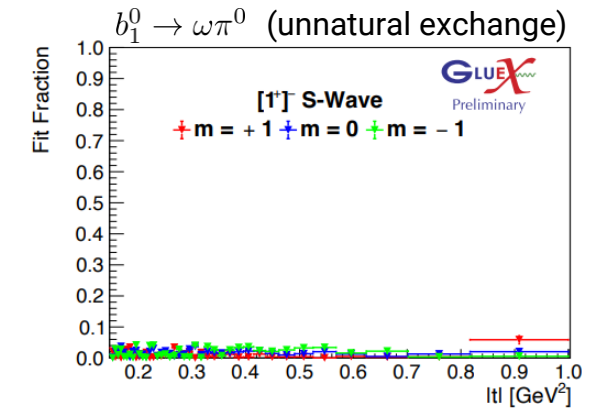
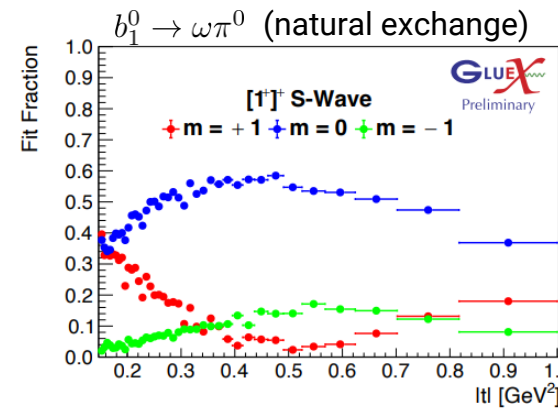
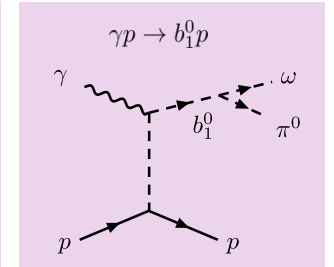
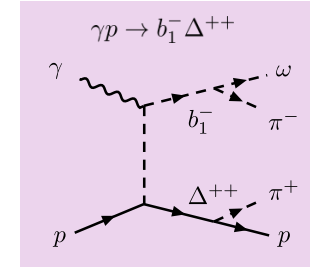
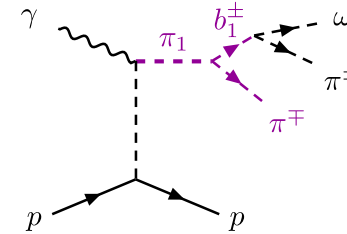
- Size of pion exchange consistent with preliminary GlueX data.



Interaction: $\langle \pi | \gamma b_1 \rangle = \frac{g_{b_1}}{m_{b_1}} \epsilon_\mu^{b_1^*} \epsilon_\nu^{\gamma^*} [p_1^\mu p_3^\nu - (p_1 \cdot p_3) g_{\mu\nu}]$

$$= \frac{g_{b_1}}{m_{b_1}} \frac{\delta_{\lambda\gamma,\Lambda}}{2} (m_{b_1}^2 - t)$$

Regge propagator: $R_\pi(\alpha, s) = \alpha' \frac{\pi}{2} \frac{1 + e^{-i\pi\alpha}}{\Gamma(\alpha + 1)} \frac{(s/s_0)^\alpha}{\sin \pi\alpha}$

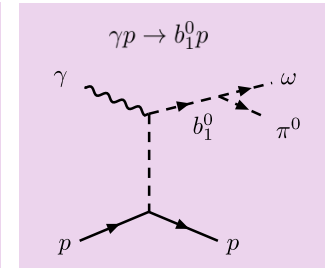
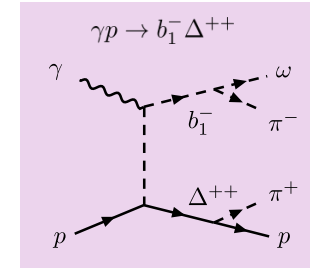
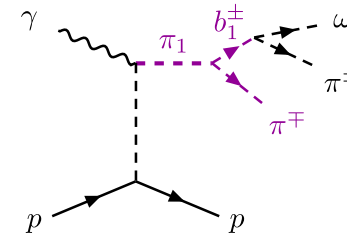


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- Derivation of cross section formula and implementation in AmpTools (GlueX analysis code).

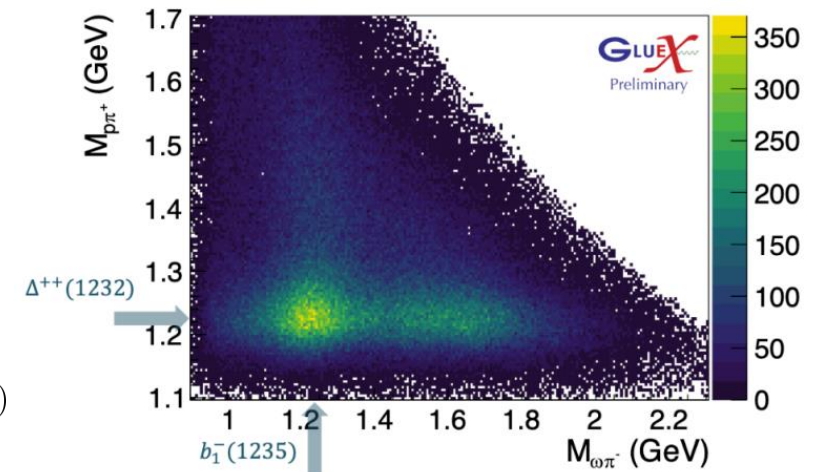
$$A_{\lambda_\gamma, \lambda_1, \lambda_2} = \sum_{i=1^\pm} \sum_{\Lambda=-1}^1 \sum_{\lambda_\Delta=-\frac{3}{2}}^{\frac{3}{2}} V_{\lambda_\gamma, \Lambda; \lambda_1, \lambda_\Delta}^i(s, t) \sum_{\lambda=-1}^1 \underbrace{F_\lambda^i D_{\Lambda, \lambda}^{J*}(\Omega_\omega) Y_\lambda^1(\Omega_H) G \tilde{F}_{\lambda_2} D_{\lambda_\Delta, \lambda_2}^{\frac{3}{2}*}(\Omega_p)}_{\text{Decay chain}}$$

Production Decay chain

$\gamma(\lambda_\gamma)p(\lambda_1) \rightarrow b_1^-(\Lambda)\Delta^{++}(\lambda_\Delta)$

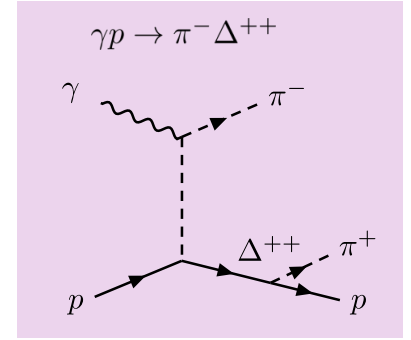
$b_1^-(\Lambda) \rightarrow \omega(\lambda)\pi^-$
 $\omega(\lambda) \rightarrow \pi^+\pi^-\pi^0$
 $\Delta^{++}(\lambda_\Delta) \rightarrow \pi^+p(\lambda_2)$

- Access SDMEs for b_1^- and Δ^{++}



SDMEs of the Δ^{++}

- Spin density matrix elements (SDMEs) of the $\Delta^{++}(1232)$ in $\gamma p \rightarrow \pi^- \Delta^{++}$ comparing with experimental data from GlueX.
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- JPAC previous model reproduces cross section but not SDMEs.



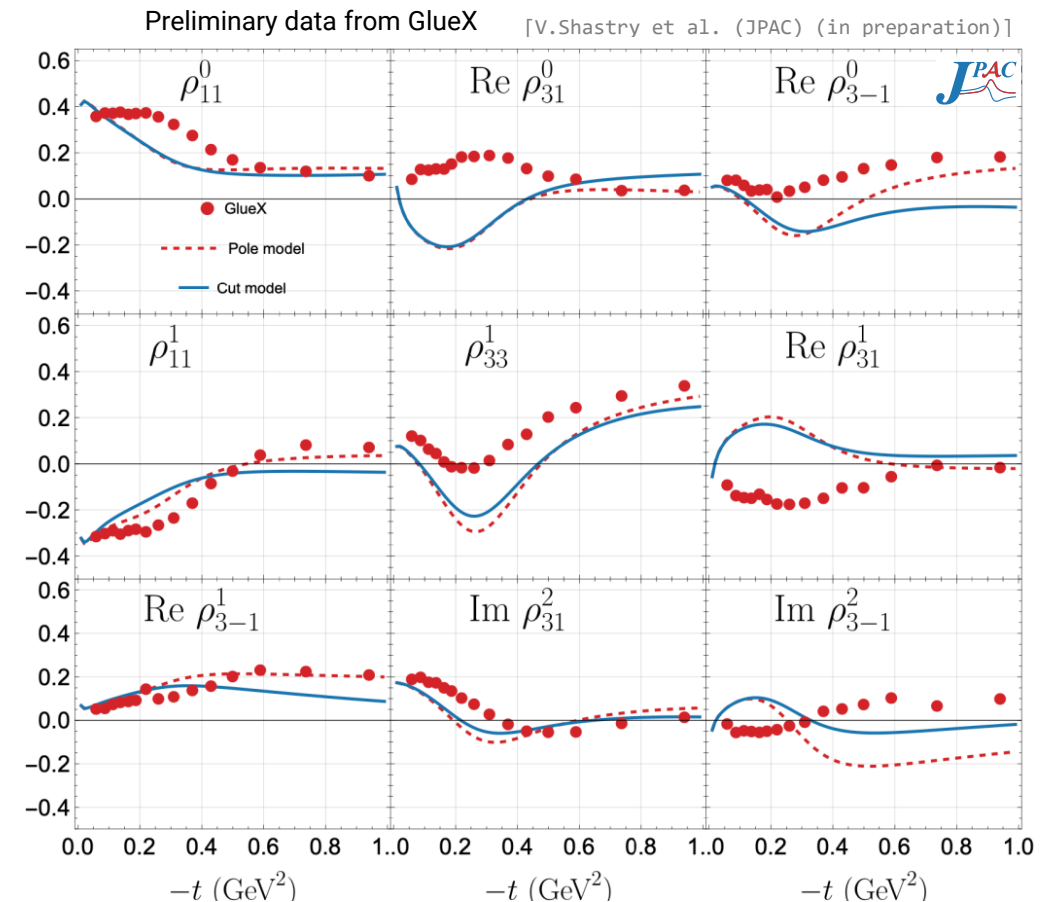
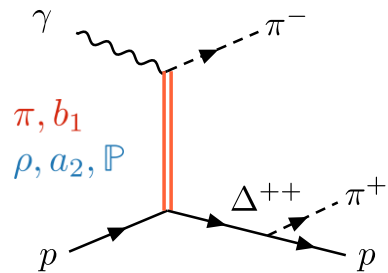
[J.Nys et al. (JPAC), *Phys.Lett.B* 779 (2018) 77-81]

$$A_{\mu\gamma\mu_1\mu_2}(s, t) = \beta_{\mu\gamma}(t)\beta_{\mu_1\mu_2}(t)\mathcal{P}_R(s, t)$$

$$\mathcal{P}_R = \frac{\pi\alpha'_R \tau_R + e^{-i\pi\alpha_R(t)}}{2 \sin \pi\alpha_R(t)} \left(\frac{s}{s_0}\right)^{\alpha_R(t)}$$

+ Poor Man's Absorption for pion exchange

[P.K.Williams, *Phys.Rev.D* 1 (1970) 1312]



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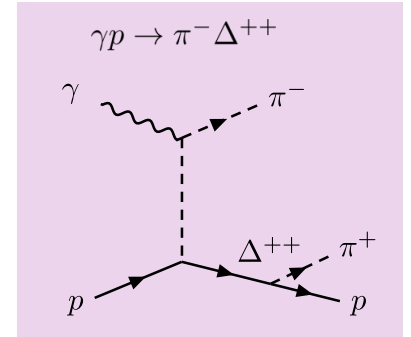
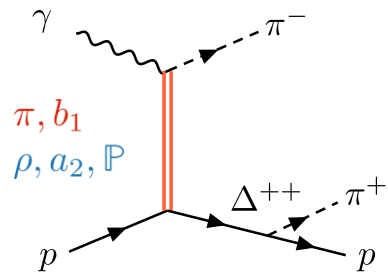
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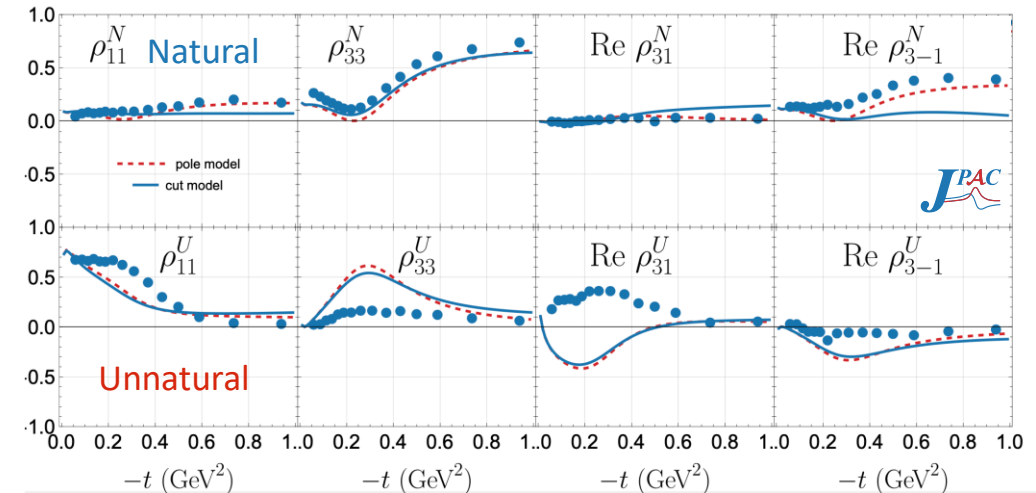
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Preliminary data from GlueX

[V.Shastry et al. (JPAC) (in preparation)]



- Better agreement with natural exchange.

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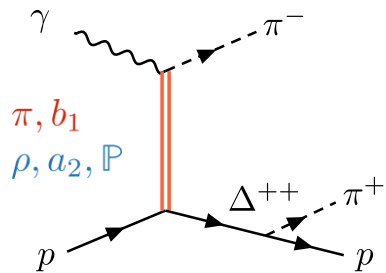
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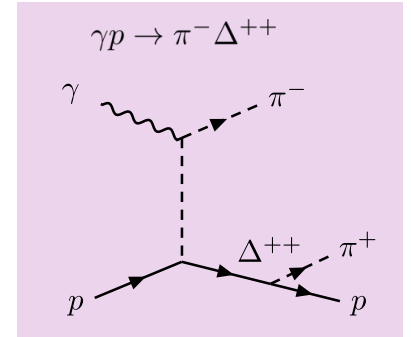
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- Better agreement with natural exchange.
- Polynomial can fit the data.

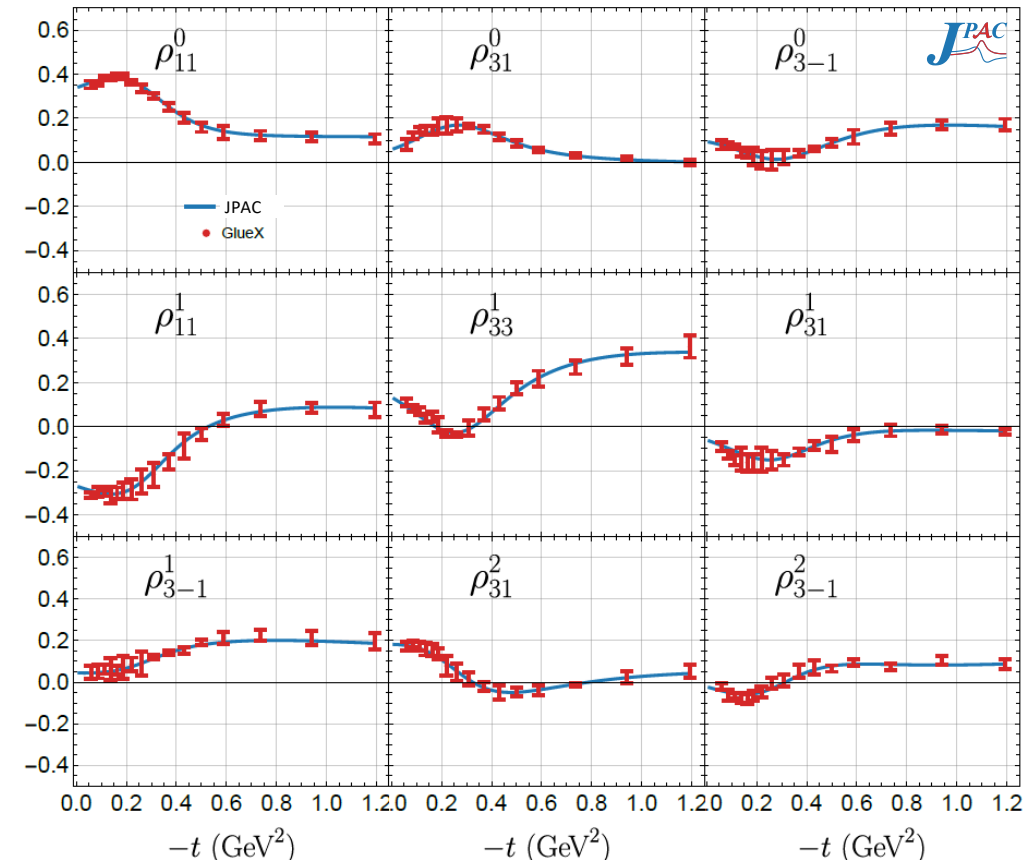
$$V_{\lambda_\gamma, \lambda_1, \lambda_\Delta}(t) = (z_{\lambda_\gamma, \lambda_1, \lambda_\Delta}^2 t^2 + z_{\lambda_\gamma, \lambda_1, \lambda_\Delta}^1 t + z_{\lambda_\gamma, \lambda_1, \lambda_\Delta}^0) e^{at}$$

- Work in progress to identify the Physics.



Preliminary data from GlueX

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CONCLUSIONS

- A precise comprehension of the production mechanisms is crucial for the light hybrid meson searches.
- At high energies, meson photoproduction reactions are dominated by the exchange of Regge trajectories, in particular, the pion trajectory plays a major role at low momentum transfer.
- How do we reggeized the pion appropriately?
 - Current conservation requires the nucleon Born terms (gauge invariance).
 - It was not clear how to add t - and s -channel consistently without double counting: t -channel and s -channel partial wave series should independently represent the full amplitude.
 - Examination of the analytical J dependence emerging from the contraction of the vertices coupling $\gamma\pi$ and NN to $J^P = (\text{even})^-$ reveals that it is analytical at $J = 0$ and physically contains part of the (s -channel, or u -channel depending on charge) nucleon exchange.

What's next?

- Revisit the pion exchange in $\gamma p \rightarrow \pi^- \Delta^{++}$ and understand Δ^{++} SDMEs.
- Extension of the formalism to natural parity exchanges.
- Amplitudes for photoproduction of b_1 , a_2 , π_1 with proton and Δ^{++} recoils.
- Close collaboration with GlueX to provide them with theory support.

